IRSTI 30.19.21 https://doi.org/10.26577/ijmph.2023.v14.i2.02

B.D. Dzhanmuldaev[1*], A.B. Janmuldayeva[2],
Zh. Jalbyrova[1], A.K. Smakhanova[1], A.Z. Madelkhanova[1]

[1]Korkytata Kyzylorda University, Kyzylorda, Kazakhstan
[2]Eurasian National University, Astana, Kazakhstan
E-Mail: Janmuldaeva1210@mail.ru
(*Received 19 November 2023; accepted 27 November 2023*)

Solution of one boundary value task of viscoelasticityin a nonlinear formulation, in the case of a cubic stress-strain relation

Abstract. In this paper, the solution of a boundary value task in the nonlinear formulation is considered by the authors [1][2]. In spite of its proximity to linear theory, the nonlinear theory of viscoelasticityhas not yet been fully developed. This issue is far from being fully completed, since the existing calculation methods do not yet provide a complete answer to the many different questions posedby practice. For this reason, in order to obtain a nonlinear law relating the strains σ_{ij} and deformations ε_{ij} a number of conditions are formed:

(1) The specific work of deformation A must be a function of the entire deformation historyfrom the beginning of deformation to the current time t.

(2) The material of a viscoelastic body is homogeneous and isotropic.

(3) For very small deformations the nonlinear relation law between σ_{ij} and ε_{ij} in the limit should pass to relations in linear approximation.

Key words: bulk compression modulus, linear integral operator, kernel of integral operator, nonlinear dependence,quadratic strain intensity, Fourier and Laplace transforms.

Introduction

Formulation of the boundary task of vibrations of isotropic plates lying on a deformable base with nonlinear stress-strain relation

Let us assume that the vibrations of the plate lying on a deformable base can be caused both by external forces on the surface of the plate and by perturbations propagating from the base. In addition, we will assume that at the contact boundaries of the plate with the base, these contacts are ideal, i.e., there is no friction.

For simplicity, consider the plate and the base in the (x,z) plane or when the external forces do not depend on the y-coordinate. In this case the displacements u_i, w_i are nonzero and the displacement $v_i = 0$, i.e. is absent.

The case is considered when the base material is isotropic and the stress-strain relation is linear, and the stress-strain relation for the plate is assumed to be cubic. In this case, the Boltzmann type relations for the base are fulfilled:

$$\sigma_{jj}^{(2)} = L_1\left(\varepsilon_{jj}^{(2)}\right) + 2M_2\left(\varepsilon_{jj}^{(2)}\right)$$

$$\sigma_{ij}^{(2)} = M_2\left(\varepsilon_{ij}^{(2)}\right), (i,j = x,z; i \neq j) \quad (1.1)$$

for the plate, the following ratios are satisfied:

$$\sigma_{jj}^{(1)} = 3K_1 R_0^{(1)}\left\{\varepsilon_0^{(1)}\left[1 + \alpha\chi_0^{(1)} K_2^{(1)}\left(\varepsilon_0^{(1)2}\right)\right]\right\} + 2G_1 R^{(1)}\left\{\varepsilon_{jj}^{(1)} - \varepsilon_0^{(1)}\right\} \cdot [1 + \alpha\gamma_0^{(1)} \cdot G_1^{(1)}(\psi_0^{(1)^2}] \quad (1.2)$$
$$\sigma_{jj}^{(1)} = G_1 R^{(1)}\left\{\varepsilon_{ij}^{(1)}\left[1 + \alpha\gamma_0^{(1)} G_1^{(1)}\left(\psi_0^{(1)^2}\right)\right]\right\} \ (i \neq j, i, j = x, z)$$

where $\varepsilon^{(1)}$is the average volumetric deformation

$\psi_0^{(1)^2}$ is the quadratic strain intensity, i.e.

$$\psi_0^{(1)^2} = \frac{2}{\sqrt{3}}\left[\frac{2}{3}\left(\varepsilon_{xx}^{(1)^2} + \varepsilon_{zz}^{(1)^2} - \varepsilon_{xx}^{(1)^2}\cdot\varepsilon_{zz}^{(1)^2} + \frac{1}{2}\varepsilon_{xz}^{(1)^2}\right)\right] \quad (1.3)$$

$\chi_0^{(1)}, \gamma_0^{(1)}$ are functions of elongation and shear, respectively, which are determined by the formulas:

$$\chi_0^{(1)} = 1 + F_0^{(1)}\left(\varepsilon_0^{(1)}\right);\ \gamma_0^{(1)}(\psi_0^{(1)^2})) = 1 + F_1^{(1)}\left(\psi_0^{(1)^2}\right);\ F_j^{(1)}(0) = 0 \quad (1.4)$$

At the same time the functions $F_0^{(1)}$и $F_1^{(1)}$ are decomposed into a degree series

$$F_0^{(1)}(\varepsilon_0^{(1)}) = \sum_{n=0}^{\infty}\alpha_n\cdot(\varepsilon_0^{(1)})^{2(n+1)}\ (1.5)\ F_1^{(1)}\left(\psi_0^{(1)^2}\right) = \sum_{n=0}^{\infty}\gamma_n(\psi_0^{(1)})^{2(n+1)}$$

$R_0^{(1)}$ and $R^{(1)}$ are linear integral operators of the Volterra type

$$R_0^{(1)}(\zeta) = \zeta(t) - \int_0^t F_{10}(t-\zeta)\zeta(\xi)d\zeta \quad (1.6)$$

$$R^{(1)}(\zeta) = \zeta(t) - \int_0^t F_{20}(t-\zeta)\zeta(\xi)d\zeta$$

$R_2^{(1)}, G_1^{(1)}$ are nonlinear viscoelastic operators

$$K_2^{(1)}\left(\varepsilon_0^{(1)^2}\right) = \varepsilon_0^{(1)^2} - \int_0^t\int_0^t F_0^{(1)}[(t-\xi_1)(t-\xi_2)]\varepsilon_0^{(1)}(\xi_2)d\xi_1 d\xi_2 \quad (1.7)$$

$$G_1^{(1)}(\psi_0^{(1)^2}) = \psi_0^{(1)^2} - \int_0^t F_1^{(1)}(t-\xi)\psi_0^{(1)^2}(\xi)d\xi$$

The constants K_1 and G_1 are equal to

$$K_1 = \lambda_1 + \frac{2}{3}\mu_1;\ G_1 = \mu_1 \quad (1.8)$$

The equations of vibration of the plate as a viscoelastic layer have the form:

$$\left(K_2^{(1)}R_0^{(1)} + \frac{4}{3}G_1R^{(1)}\right)\frac{\partial^2 u_1}{\partial x^2} + G_1R^{(1)}\frac{\partial^2 u_1}{\partial z^2} + \left(K_1R_0^{(1)} + \frac{1}{3}G_1R^{(1)}\right)\frac{\partial^2 w_1}{\partial x\partial z} + \alpha F_1^{(1)}(u, w_1) = \rho_1\frac{\partial^2 u_1}{\partial t^2} \quad (1.9)$$

$$\left(\mathrm{K}_1R_0^{(1)} + \frac{1}{3}G_1R^{(1)}\right)\frac{\partial^2 u_1}{\partial x\partial z} + G_1R^{(1)}\frac{\partial^2 w_1}{\partial z^2} + \left(K_1R_0^{(1)} + \frac{4}{3}G_1R^{(1)}\right)\frac{\partial^2 w_1}{\partial z^2} + \alpha F_2^{(1)}(u_1, w_1) = \rho_1\frac{\partial^2 w_1}{\partial t^2} \quad (1.9)$$

where $F_1^{(1)}, F_2^{(1)}$ are nonlinear operators.

$$F_1^{(1)}(u_1, w_1) = 3k_1\lambda_0^{(1)}R_0^{(1)}\left\{\frac{\partial}{\partial x}\left[\varepsilon_0^{(1)}K_2^{(1)}\left(\varepsilon_0^{(1)^2}\right)\right]\right\} +$$

$$+\gamma_0\left\{G_2R^{(1)}\frac{\partial}{\partial x}\left[\left(\varepsilon_{xx}^{(1)} - \varepsilon_0^{(1)}\right)G_1^{(1)}(\psi_0^{1^2})\right]\right\} + \gamma_0G_1R^{(1)}\frac{\partial}{\partial x}\left[\varepsilon_{xz}^{(1)}G_1^{(1)}(\psi_0^{1^2})\right] \quad (1.10)$$

$$F_2^{(1)}(u_1, w_1) = 3\kappa_1\chi_0^{(1)}R_0^{(1)}\left\{\frac{\partial}{\partial z}\left[\varepsilon_0^{(1)}K_2^{(1)}\left(\varepsilon_0^{(1)^2}\right)\right]\right\} +$$

$$+\gamma_0\left\{G_1R_1\frac{\partial}{\partial z}\left[\left(\varepsilon_{xx}^{(1)} - \varepsilon_0^{(1)}\right)G_1^{(1)}(\psi_0^{1^2})\right]\right\} + \gamma_0G_1R^{(1)}\frac{\partial}{\partial x}\left[\varepsilon_{xz}^{(1)}G_1^{(1)}(\psi_0^{1^2})\right]$$

Boundary conditions:

$$\text{at } z = h, \sigma_{zz}^{(1)} = f_z^{(1)}(x,t);\ \sigma_{xz} = 0 \quad (1.11)$$

$$\text{at } z = -h, \sigma_{zz}^{(1)} = \sigma_{zz}^{(2)};\ \sigma_{xz}^{(1)} = 0;\ \sigma_{xz}^{(2)} = 0;\ w_1 = w_2$$

The initial conditions are zero, i.e. $u_l = \frac{\partial u_l}{\partial t} = w_l = \frac{\partial w_l}{\partial t} = 0\ at\ t = 0.$ (1.12)

Thus, the boundary task of vibrations of isotropic plates lying on a deformable base taking into account the physical nonlinearity of the stress from deformation is reduced to the solution of the integro-differential equations (1.9) at boundary and initial conditions (1.11)-(1.12).

2. General and based on them approximate equations of vibration of a viscoelastic plate lying on a deformable base in the nonlinear formulation

If relations (1.2) are satisfied for the plate material, then the displacements u and w of points of the plate will be searched for in the form of a series on the parameter α.

$$u(x,z,t) = \sum_{n=0}^{\infty} \alpha^n u_n(x,z,t)$$
$$w(x,z,t) = \sum_{n=0}^{\infty} \alpha^n w_n(x,z,t) \quad (2.1)$$

In this case, the parameter α will be considered small, i.e. nonlinearity is considered to be weak. If we limit ourselves to the first two summands in the series (2.1), then for $u_0, w\ and\ u_1, w_1$ we have the equations:

$$L_1\left(\frac{\partial^2 u_0}{\partial x^2}\right) + M_1\left(\frac{\partial^2 u_0}{\partial z^2}\right) + (L_1 + M_1)\left(\frac{\partial^2 w_0}{\partial x \partial z}\right) = \rho_1 \frac{\partial^2 u_0}{\partial t^2}$$
$$(L_1 + M_1)\left(\frac{\partial^2 u_0}{\partial x \partial z}\right) + M_1\left(\frac{\partial^2 w}{\partial x^2}\right) + L_1\left(\frac{\partial^2 w_0}{\partial z^2}\right) = \rho_1 \frac{\partial^2 w_0}{\partial t^2} \quad (2.2)$$

$$L_1\left(\frac{\partial^2 u_1}{\partial x^2}\right) + M_1\left(\frac{\partial^2 u_1}{\partial z^2}\right) + (L_1 + M_1)\left(\frac{\partial^2 w_1}{\partial x \partial z}\right) + F_1(u_0, w_0) = \rho_1 \frac{\partial^2 u_1}{\partial t^2}$$
$$(L_1 + M_1)\left(\frac{\partial^2 u_1}{\partial x \partial z}\right) + M_1\left(\frac{\partial^2 w_1}{\partial x^2}\right) + L_1\left(\frac{\partial^2 w_1}{\partial z^2}\right) + F_2(u_0, \omega_0) = \rho_1 \frac{\partial^2 w_1}{\partial t^2} \quad (2.3)$$

thus the task is reduced to a system of two linear tasks. Task (2.2) with boundary conditions (1.11) and (1.12), is a task of vibration of a plate lying on a deformable base in the flat setting, so it will be considered solved.

For equations (2.3) the boundary conditions look like:

at $z = h, \sigma_{zz}^{(1)} = 0;\ \sigma_{xz}^{(1)} = 0$ (2.4)

at $z = -h, \sigma_{zz}^{(1)} = R(w_1);\ \sigma_{xz}^{(1)} = 0$ (2.5)

where the operator R is found after the expression is inverted

$$R_0 = \frac{(\beta^2 + \kappa^2 + q^2)^2 - 4\alpha_2\beta_2(\kappa^2 + q^2)}{\alpha^2(\beta_2^2 - \kappa^2 - q^2)} \quad (2.6)$$

on k,q,p (k and q are Fourier transform parameters, p is the Laplace transform parameter).

As can be seen from the boundary conditions (1.12) at $z = -h$ the base parameters and R the base reaction are excluded. Thus, we have the task (2.3) of vibrations of isotropic plate under boundary conditions (2.4) and (2.5) taking into account physical nonlinearity of stresses from deformation.

In this formulation, the left-hand sides of equation (2.3) have nonlinear terms $F_1(u_0, w_0)$ and $F_2(u_0, w_0)$ that depend on displacements u_0, w_0 and look like (1.10).

Then, applying integral Fourier and Laplace transforms to the displacements u_1, w_1, as well as non-linear functions $F_1(u_0, w_0)$, $F_2(u_0, w_0)$ we obtain the ordinary differential equations

$$\mathrm{M}_{10}\frac{d^2u_{10}}{dz^2} - [\rho_1 p^2 + k^2 L_{10}]u_{10} - k[L_{10} + M_{10}]\frac{dw_{10}}{dz} = F_{10}(u_0, w_0)$$
$$L_{10}\frac{d^2w_{10}}{dz^2} - [\rho_1 p^2 + k^2 M_{10}]w_{10} - k[L_{10} + M_{10}]\frac{du_{10}}{dz} = F_{20}(u_0, w_0) \quad (2.7)$$

where F_{10} и F_{20} are Fourier and Laplace transformed nonlinear functions $F_1(u_0, w_0)$, $F_2(u_0, w_0)$.

$$F_{10} = \int_0^\infty \left.\begin{matrix} sinkx \\ -coskx \end{matrix}\right\} dk \int_l F_1 \exp(pt)\, dt$$

$$F_{20} = \int_0^\infty \left.\begin{matrix} coskx \\ sinkx \end{matrix}\right\} dk \int_l F_2 \exp(pt)\, dp$$

General solutions of equations (2.7) we find in the form

$$U_{10} = k[A_1 ch(\alpha z) + B_1 sh(\alpha z)] + \beta[A_2 ch(\beta z) + B_2 sh(\beta z)] - \frac{1}{\alpha(\beta^2 - \alpha^2)} \cdot \int_0^z F(\xi) sh[\alpha(z-\xi)]d\xi + \frac{1}{\beta(\beta^2 - \alpha^2)} \int_0^z F(\xi) sh[\beta(z-\xi)]\, d\xi \quad (2.8)$$

$$w_{10} = -\alpha[A_1 sh(\alpha z) + B_1 ch(\alpha z)] - k[A_2 sh(\beta z) + B_2 ch(\beta z)] + \frac{1}{k(\beta^2 - \alpha^2)} \cdot \int_0^z F(\xi) ch[\alpha(z-\xi)]d\xi - \frac{k}{\beta^2(\beta^2 - \alpha^2)} \int_0^z F(\xi) ch[\beta(z-\xi)]d\xi$$

where

$$F(z) = \frac{k(L_{10} + M_{10})}{L_{10} \cdot M_{10}} \cdot \frac{dF_{20}}{dz} + \frac{1}{L_{10}} \cdot \frac{d^2F_{10}}{dz^2} - \frac{\beta^2}{L_{10}} \cdot F_{10}$$

In this case the function F(z) is assumed to be given, and the integrals $\int_0^z ch[\gamma(z-\xi)]d\xi$ и $\int_0^z sh[\gamma(z-\xi)]d\xi$ can be expanded into power series.

Decomposing the expressions for u_{10} and w_{10} into power series on the coordinate z and entering the principal parts of the displacements by the formulas:

$$U_{10} = kA_1 + \beta A_2 \; U_{10}^{(1)} = kB_1\alpha + \beta^2 B_2$$
$$W_{10}^{(1)} = -\alpha^2 A_1 - k\beta A_2; \; W_0^{(1)} = -\alpha B_1 - kB_2$$

and reversing by k and β, we get:

$$u_1 = \sum_{n=0}^\infty \left\{\left[\lambda_1^{(n)} - \lambda_1^{(1)} c_t Q_n\right] U_1 + c_t Q_n \frac{\partial W_1^{(1)}}{\partial x} + F_{2n}^{(1)}\right\} \frac{z^{2n}}{(2n)!} + \sum_{n=0}^\infty \left\{\left[\lambda_2^{(n)} - \frac{\partial^2}{\partial^2 x} D_1 Q_n\right] U_1^{(1)} + D_1 Q_n \frac{\partial}{\partial x} \lambda_2^{(1)} W_1^{(1)} + F_{2n+1}^{(2)}\right\} \frac{z^{2n+1}}{(2n+1)!}$$

$$w_1 = \sum_{n=0}^\infty \left\{\left[c_1 \lambda_1^{(1)} Q_n\right] \frac{\partial U_1}{\partial x} + \left[\lambda_1^{(n)} - c_t \frac{\partial^2}{\partial^2 x} Q_n\right] W_1^{(1)} + F_{2n}^{(3)}\right\} \frac{z^{2n+1}}{(2n+1)!} + \sum_{n=0}^\infty \left\{-D_1 Q_n \frac{\partial U_1^{(1)}}{\partial x} + \left[\lambda_2^{(n)} - \lambda_2^{(1)} D_1 Q_n\right] W^{(1)} + F_{2n+1}^{(n)}\right\} \frac{z^{2n}}{(2n)!}$$

where

$$F_{2n}^{(1)} = F\left[\frac{\beta^2 - k^2}{k(\beta^2 - \alpha^2)} + \cdots + \frac{\alpha^{2n}(\beta^2 + k^2) - 2k^2\beta^{2n}}{k(\beta^2 - \alpha^2)}\right]$$

$$F_{2n+1}^{(2)} = F\left[\frac{\beta^2 - k^2}{\beta^2(\beta^2 - \alpha^2)} + \cdots + \frac{2\beta^{2(n+1)} - (\beta^2 + k^2)\beta^{2n}}{\beta^2(\beta^2 - \alpha^2)}\right]$$

$$F_{2n}^{(3)} = \frac{\partial}{\partial z}F_{2n}^{(1)}\Big|_{z=0};\ F_{2n+1}^{(4)} = \frac{\partial}{\partial z}F_{2n+1}^{(2)}\Big|_{z=0}$$

Then from the boundary conditions (2.4) and (2.5) we obtain a system of four equations with respect to $U_1, U_1^{(1)}, W_1^{(1)}$ and $W^{(1)}$.

$$M'_{1(n)}(U_1) + M'_{2(n)}\left(W_1^{(1)}\right) + M'_{3(n)}\left(U_1^{(1)}\right) + M_{4(n)}\left(W^{(1)}\right) = M_{5(n)}\left(F_{2n}^{(1,3)}\right)$$

(2.10)

$$K'_{1(n)}(U_1) + K'_{2(n)}\left(W_1^{(1)}\right) + K'_{3(n)}\left(U_1^{(1)}\right) + K'_{4(n)}\left(W^{(1)}\right) = K'_{5(n)}\left(F_{2n}^{(1,3)}\right)$$

$$D_{1(n)}^{(R)}(U_1) + D_{2(n)}^{(R)}\left(W_1^{(1)}\right) + D_{3(n)}^{(R)}\left(U_1^{(1)}\right) + D^R{}_{4(n)}\left(W^{(1)}\right) = D_{5(n)}\left(F_{2n}^{(i)}, F_{2n+1}^{(i)}\right)$$

$$-K'_{1(n)}(U_1) - K'_{2(n)}\left(W_1^{(1)}\right) + K'_{3(n)}\left(U_1^{(1)}\right) + K'_{4(n)}\left(W^{(1)}\right) = -K'_{5(n)}\left(F_{2n+1}^{(2,4)}\right)$$

The system of equations (2.10) are equations of longitudinal-transverse vibration of the plate in a nonlinear formulation, lying on a deformable base in the first approximation.

To solve practical tasks instead of exact equations, it is advisable to use approximate equations that include some finite order on derivatives: such approximate equations are not difficult to obtain from exact ones, being limited to a finite number of the first terms. Then for the main part of the displacement $W^{(1)}$ we get an approximate equation.

$$\rho_1\frac{\partial^2 W^{(1)}}{\partial t^2} + \frac{h^2}{6}\left[\rho_1(N_1^{-1} + 3M_1^{-1}) - 4\rho_1(3 - 2M_1N_1^{-1})\frac{\partial^4 W^{(1)}}{\partial t^2\partial x^2} + 8M_1(1 - M_1N_1^{-1})\frac{\partial^4 W^{(1)}}{\partial t^4}\right] +$$
$$+\mathcal{P}\left(W^{(1)}\right) + \left[\frac{\partial F_5(W^{(1)})}{\partial x}\right]_{z=0} - \frac{h^2}{2}\left[\frac{\partial^2 F_1(W^{(1)})}{\partial x\partial z}\right]_{z=0} = 0 \quad (2.11)$$

$$\text{where } F_5 = \gamma_0 GR\left[\frac{\partial w^{(1)}}{\partial x}G_1(\psi_0^2)\right] \quad (2.12)$$

the operator $\mathcal{P}$ looks like:

$$\mathcal{P} = \frac{S}{2h}\rho_1\left\{\frac{\partial}{\partial t} + \frac{h^2}{2}\left[\rho_1(M_1^{-1} + 3L_1^{-1})\frac{\partial^3}{\partial t^3} - 4\frac{\partial^3}{\partial t\partial x^2}\right]\right\} \quad (2.13)$$

The function F_1 looks like (1.10).

For the main part of the move $W = W_0^{(1)} + \alpha W^{(1)}$ we obtain an approximate equation for the transverse displacement of the median plane of the plate lying on the deformable base in the nonlinear formulation

$$\rho_1 \frac{\partial^2 W}{\partial t^2} + \frac{h^2}{6}\left[\rho_1^2 (N_1^{-1} + 3M_1^{-1}) \frac{\partial^4 W^4}{\partial t^4} - 4\rho_1 (3 - 2M_1 N_1^{-1}) \frac{\partial^4 W}{\partial t^2 \partial x^2} + 8M_1 (1 - M_1 N_1^{-1}) \frac{\partial^4 W}{\partial x^4}\right] +$$
$$+ \mathcal{P}(W) + \alpha \left[\frac{\partial F_5(W)}{\partial x}\right]_{z=0} - \frac{h^2}{2} \alpha \left[\frac{\partial^2 F_1 W}{\partial x \partial z}\right]_{z=0} = \frac{f_z(x,t)}{h} \quad (2.14)$$

If the nonlinear dependence does not depend on the stress intensity, i.e. the parameter $\gamma_0 = 0$, then the results obtained are greatly simplified.

Of great theoretical and applied interest is the task of the effect of a normal load on the surface of an elastic plate lying on an absolutely rigid half-space with perfect contact between them.

As above, it is assumed that the stress-strain relationships are nonlinear (physical nonlinearity)

Because of the ideal contact, the sought values of displacements of points of the plate are symmetric with respect to the displacement of u and antisymmetric with respect to the displacement of v

This task is equivalent to the task about the influence of normal loads on the surface of the plate at $z = \pm h$, i.e.

$$\sigma_{zz} = f_z(x,t);\ \sigma_{xz} = 0\ (z = h) \quad (2.15)$$
$$\sigma_{zz} = w = 0\ (z = -h)$$

Consequently, this task is reduced to the study of the longitudinal vibration of a plate with a thickness of $2h$ ($|z| \le h$) based on general equations (2.10), taking as the main unknown the value of *U* points of the median plane of the plate, to determine it we obtain an approximate equation in partial derivatives

$$\frac{\partial^2 U}{\partial t^2} - c^2 \frac{\partial^2 U}{\partial x^2} - \frac{2}{9} c^2 \frac{\alpha_0 \chi_0}{\rho} \left(\frac{4\mu - c^2 \rho}{4\mu}\right) \frac{\partial}{\partial x} \left[\left(2 \frac{\partial U}{\partial x} + \frac{1}{\mu} f_z\right)^3\right] = \frac{c^2}{\rho} \left(\frac{\rho c^2}{2\mu} - 1\right) \frac{\partial f_z}{\partial x} \quad (2.16)$$

Let's consider a particular task when the magnitude of the external load $f_z(x,t)$ is equal to

$f_z(x,t) = f_z(x + Dt)$ (2.17) i.e., the task of the influence of the moving load on the plate surface at $z = \pm h$, where D is the moving speed of the moving load.

Since there are no initial conditions in the given task, it is easier to search for a general solution of equation (2.16) by passing to the moving coordinates related to the stationary coordinate system by the well-known Galileo transformation $x' = x + Dt$

Then equation (2.16) turns into the ordinary differential equation

$$(D^2 - C^2) \frac{d^2 U}{dx^2} - \frac{2}{3} c^2 \rho^{-1} \chi_0 K \left(\frac{4\mu - c^2}{4\mu}\right) \frac{d}{dx} \left[\left(2 \frac{dU}{dx} + \frac{1}{\mu} f_z\right)^3\right] = \frac{c^2}{\rho} \left(\frac{\rho c^2}{2\mu} - 1\right) \frac{df_z}{dx} \quad (2.18)$$

Equation (2.18) after integration over *x* can be reduced to a cubic equation with respect to $\frac{dU}{dx}$

$$\left(\frac{dU}{dx}\right)^3 + \frac{3}{2\mu} f_z \left(\frac{dU}{dx}\right)^2 + \left[\frac{3}{2\mu} f_z^2 - \frac{9\rho(D^2 - C^2)}{16\alpha\chi_0 K} \left(\frac{4\mu}{4\mu - \rho c^2}\right)^4\right] \frac{dU}{dx} + \left[\frac{1}{8\mu^3} f_z^3 - \frac{9(2\mu - \rho c^2)}{32\alpha\mu\chi_0 K} \left(\frac{4\mu}{4\mu - \rho c^2}\right)^4 f_z\right] = 0 \quad (2.19)$$

By substituting $\frac{dU}{dx} = S - \frac{a}{3}$ equation (2.19) is reduced to the form

$$S^3 + pS + q = 0 \qquad (2.20)$$

where $a = \frac{3}{2\mu} f_z$; $p = -\frac{9\rho(D^2-c^2)}{16\alpha\mu\chi_0 K}\left(\frac{4\mu}{4\mu-\rho c^2}\right)^4$ i.e. the parameter p does not depend on the external load, and q is equal to

$$q = \frac{9(2\rho D^2 - 2\mu)}{32\alpha\mu\chi_0 K}\left(\frac{4\mu}{4\mu-\rho c^2}\right)^4$$

Let us consider the case when $\alpha < 0$ (The case $\alpha > 0$ is solved in a similar way)

If the exposure mode is supersonic, i.e. $D > 0$ and there is no vibration in the plate in front of the load, then $\rho > 0$ and the cubic equation (2.20) has one valid solution, and the other two have no mechanical sense.

Since the nonlinearity parameter α is assumed to be small, we obtain formulas for the value of U and the loads σ_{xx}, σ_{zz} and σ_{xz} that are convenient for calculations:

$$U = \frac{1}{\rho(D^2-c^2)}\left(\frac{\rho c^2}{2\mu} - 1\right)\int_0^{x+Dt} f_z(\xi)d\xi + \alpha\left\{\frac{2}{9}\frac{\chi_0 K}{\rho(D^2-c^2}\left(\frac{4\mu-\rho c^2}{4\mu}\right)^4 \cdot \left[\frac{\rho D-4\mu}{\rho\mu(D^2-c^2}\right]^3\right\}\int_0^{x+Dt} f_z^3(\xi)d\xi \quad (2.21)$$

$$\sigma_{xx} = \frac{D^2}{D^2-c^2}\left(\frac{\rho c^2}{2\mu} - 1\right)f_z + \alpha\left\{\frac{2}{9}\cdot\frac{\chi_0 K c^2}{(D^2-c^2)}\left(\frac{4\mu - pc^2}{D^2-c^2}\right)^4\left[\frac{\rho D^2 - 2\mu}{\rho\mu(D^2-c^2}\right]^3 f_z^3 + 2(1 - \frac{\rho c}{4\mu} f_z(U)\right\} +$$
$$+\frac{z^2}{2}\left\{\left[\rho D^2\left(\frac{pc^2}{2\mu}+1\right) - \rho D^2\left(\frac{\rho c^2}{2\mu} - 1\right)^3 + 3(\lambda+2\mu)\cdot\left(\frac{\rho c^2}{2\mu} - 2\right) - 2(\rho c^2 - 3\mu)\right]\frac{\partial^3 U}{\partial x^3} - \frac{1}{\lambda+2\mu}\left[pD^2\left(\frac{\rho c^2}{2\mu} - 1\right) + 4\mu - 3(\lambda+2\mu)\right]\cdot\frac{\partial^2 f_z(U)}{\partial x^2} - \alpha\frac{4\mu}{4\mu-pc^2}\cdot\frac{\partial^2 f_z(U)}{\partial x^2}\right\}$$

$$\sigma_{zz} = f_z + \frac{z^2-h^2}{2}\left\{\frac{\rho c^2}{2\mu}(\rho D^2 - 2\mu)\frac{\partial^3 U}{\partial x^3} - \left(\frac{\rho D^2}{\lambda+2\mu} + \frac{\rho c^2}{2\mu} - 1\right)\cdot\frac{\partial^2 f_z}{\partial x^2} + \alpha\left(\frac{\rho D^2}{\lambda+2\mu} + \frac{\rho c^2}{2\mu} - 2\right)\frac{\partial^2 f_z(U)}{\partial x^2}\right\}$$

$$\sigma_{xz} = \alpha(z - \frac{1}{h^2}z^3)\frac{\chi_0 K}{192}\left(\frac{4\mu-\rho c^2}{\mu}\right)^4\left[\frac{\rho D^2 - 2\mu}{\mu\rho(D^2-c^2)}\right]^3 f_z^2 f_{zx}$$

$$f_z(U) = \frac{1}{9}\chi_0 K\left(\frac{4\mu-\rho c^2}{4\mu}\right)^3\left(2\frac{\partial U}{\partial x} + \frac{1}{\mu}f_z\right)^3$$

Let's consider a particular kind of external load function

$$f_z(x+Dt) = \sigma_0\beta(x+Dt)\exp[-\beta(x+Dt) \quad (2.22)$$

then, for example, with respect to the dimensionless load $\frac{\sigma_{xx}}{\sigma_0}$ we get an approximate expression

$$\frac{\sigma_{xx}}{\sigma_0} = l_1\xi\exp(-\xi) + \alpha l_2\sigma_0^2\xi^3\exp(-3\xi) \quad (2.23)$$

where the following designations are used

$$l_1 = \frac{D^2}{D^2-c^2}\left(\frac{pc^2}{2\mu} - 1\right) > 0;$$

$$l_2 = \frac{D^2}{D^2-c^2}\cdot\frac{\chi_0 K(4\mu-\rho c^2)^4(\rho D^2-2\mu)^3}{1152\mu^7\rho^3(D^2-c^2)}$$

When $\alpha < 0$ in a nonlinear task, the maximum value of $\frac{\sigma_{xx}}{\sigma_0}$ is less than in a linear task. When $\alpha > 0$ there is the opposite phenomenon

$$\sigma = \frac{l_2}{l_1} + \alpha\frac{c_0^2 l_2}{l_1^3}\ ;\ \sigma_2 = \frac{l_2}{l_1}$$

Conclusion

This article gives a general formulation of the boundary value task of vibrations of isotropic plates lying on a deformable base in a nonlinear formulation. It is shown that the boundary task of vibrations of isotropic plates lying on a deformable base considering physical nonlinearity of stresses from deformation is reduced to the solution of integrodifferential equations with given boundary and initial conditions.

The general equations of oscillation of isotropic flat structures lying on a deformable base, taking into account the physical nonlinearity of stresses from deformation, are obtained.

It is shown that the general equations of vibration of isotropic plates, considering the physical nonlinearity of stresses from deformations, are complex in structure and contain derivatives of any order for the coordinates x, y and time t, and therefore are not suitable for practical tasks and engineering calculations.

To solve practical tasks, approximate equations that involve some finite order on derivatives are derived. The approximate equations are derived from the exact equations by limiting themselves to a finite number of the first terms.

The task of normal loading on the surface of an elastic plate lying on an absolutely rigid half-space with nonlinear dependence of stresses on deformations is considered. Taking into account the small nonlinearity parameter, formulas for displacement and for stresses convenient for calculations are obtained.

A particular type of external load is considered and an approximate expression for the dimensionless stress is obtained.

References

1. Adamov, A.A., Dzhanmuldaev, B.D., Janmuldayeva, A.B. (2020). Application of one mathematical method in solving boundary value problems of natural vibrations of a rectangular viscoelastic plate located under the surface of a deformable medium. Journal of Physics: Conference Seriesthis link is disabled,, (2020), 1425(1), 012022
2. Temirbekov A.N., Temirbekova L.N., Zhumagulov B.T. Fictitious domain method with the idea of conjugate optimization for non-linear Navier-Stokes equations. Applied and Computational Mathematics. –2023.-Vol. 22, Issue 2.-P.172–188. https://doi.org/10.30546/1683-6154.22.2.2023.172 (Scopus procentile – 98, SJR=1.191, Scopus quartile – Q1, Web of Science quartile – Q1, IF=10).
3. Urgenishbekov, A.T., Dzhanmuldaev, B.D.(2004)., Elaboration of a linear theory of thermoviscous-elastic plate oscillations. Promyshlennoe i Grazhdanskoe Stroitel'stvo, (2004), (4), pp. 48–50
4. Dzhanmuldaev, B.D. Oscillations of flat structures laid on base subjected to deformation taking into account anisotropy and prestressing. Promyshlennoe i Grazhdanskoe Stroitel'stvo,(2003), (9), pp. 40–45
5. Dzhanmuldaev, B.D., Filippov, I.G. Oscillations of isotropic plates, placed on a deformable base in non-linear arrangement. Promyshlennoe i Grazhdanskoe Stroitel'stvo,(2002), (12), pp. 28–29
6. Dzhanmuldaev, B.D. Dynamic interaction of flat isotropic members and the surrounding deformative medium taking into account the temperature. Promyshlennoe i Grazhdanskoe Stroitel'stvo, (2002), (11), pp. 43–44
7. Vidana Pathiranagei, S. , Gratchev, I. Coupled thermo-mechanical constitutive damage model for sandstone. (2022), Journal of Rock Mechanics and Geotechnical Engineering 14(6), pp. 1710-1721
8. Luo, J., He, J. Constitutive Model and Fracture Failure of Sandstone Damage under High Temperature–Cyclic Stress. (2022), Materials 15(14),4903
9. Arias-Buitrago, J.A., Alzate-Espinosa, G.A., Arbelaez-Londoño, A., Zambrano-Narvaez, G., Chalaturnyk, R., Experimental study on the effect of temperature on the mechanical properties of unconsolidated silty sandstones., (2021), Energies 14(21),7007
10. Morales-Monsalve, C.B., Arbelaez-Londoño, A., Alzate-Espinosa, G., Araujo-Guerrero, E.F. Effect of the stress path in the mechanical behavior of unconsolidated sands,(2020), ISRM VIII Brazilian Symposium on Rock Mechanics, SBMR 2018
11. Morales-Monsalve, C.B., Alzate-Espinosa, G.A., Arbelaez-Londoño, A., Stress–Strain Behavior in Heavy Oil Reservoirs Using Mohr–Coulomb Constitutive Model,(2019), Geotechnical and Geological Engineering 37(4), pp. 3343-3353
12. Morales-Monsalve, C.B., Lara-Restrepo, I.F., Araujo-Guerrero, E.F., Alzate-Espinosa, G.A., Arbelaez-Londoño, A., Naranjo-Agudelo, A. Effect of Temperature on the Strength Parameters at the Plastic Domain for Unconsolidated Sandstones.(2018), *Geotechnical and Geological Engineering*, 36 (6), pp. 3537-3549
13. Yao, M., Rong, G., Zhou, C., Peng, J. Effects of thermal damage and confining pressure on the mechanical properties of coarse marble(2016), *Rock Mechanics and Rock Engineering*, 49 (6), pp. 2043-2054.
14. Zhang, P., Mishra, B., Heasley, K.A. Experimental Investigation on the Influence of High Pressure and High Temperature on the Mechanical Properties of Deep Reservoir Rocks,(2015), *Rock Mechanics and Rock Engineering*, 48 (6), pp. 2197-2211
15. Araujo, R.G.S., Sousa, J.L.A.O., Bloch, M., Experimental investigation on the influence of temperature on the mechanical properties of reservoir rocks, (1997) *International journal of rock mechanics and mining sciences & geomechanics abstracts*, 34 (3-4), p. 459.
16. Benzagouta, M.S., Amro, M.M. Pressure and temperature effect on petrophysical characteristics: Carbonate reservoir case.(2009) *Society of Petroleum Engineers – SPE Saudi Arabia Section Technical Symposium 2009*.