Effects of the axion through the Higgs portal on primordial gravitational waves during the electroweak breaking

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(Received 3 January 2023; accepted 21 March 2023; published 30 March 2023)

We investigate the effects of short axion kination eras on the energy spectrum of the primordial gravitational waves corresponding to modes that reenter the Hubble horizon at the postelectroweak symmetry breaking epoch well within the radiation domination era. Our main assumption is the existence of an extremely weakly coupled hidden sector between the Higgs and the axion, materialized by higher order nonrenormalizable dimension six and dimension eight operators, active at a scale M of the order 20–100 TeV. This new physics scale M, which is way higher than the electroweak scale, is motivated by the lack of new particle observations in the large hadron collider to date beyond the electroweak scale. Once the electroweak symmetry breaking occurs at $T \sim 100$ GeV, the axion potential acquires a new minimum due to the new terms generated by the electroweak breaking, and the axion oscillations at the origin are destabilized. In effect after some considerable amount of time, the axion rolls swiftly to its new minimum, experiencing a short kination epoch, where its energy density redshifts as $\rho_a \sim a^{-6}$. After it reaches the new minimum, since the latter is energetically less favorable that the Higgs minimum, it decays to the Higgs minimum and the Universe is described again by the Higgs minimum. The axion returns to the origin and commences again oscillations initiated by quantum fluctuations, redshifts as dark matter, and the same procedure is repeated perpetually. These short axion kination eras may disturb the background total equation of state parameter during the radiation domination era, changing it from that of radiation $w = 1/3$ to some value closer to the kination value $w = 1$. We examined the effect of a value $w = 1/2$ on the energy density of the primordial gravitational waves. As we show, the energy spectrum of the gravitational waves mainly depends on how many times the short axion kination epochs occur, on the inflationary theory, on the actual value of the background equation of state parameter during the short kination eras, and finally on the reheating temperature. Our findings indicate a characteristic shape in the energy spectrum that can be observed in future gravitational wave experiments. We however disregarded the contribution of the electroweak phase transition on the gravitational waves for simplicity and transparency of our results.

DOI: 10.1103/PhysRevD.107.064071

I. INTRODUCTION

The focus in contemporary scientific research related to theoretical and particle physics is undoubtedly in the sky and specifically on gravitational wave experiments. The stage 4 cosmic microwave background (CMB) experiments [1,2] and the future gravitational wave experiments such as LISA, BBO, DECIGO, and the Einstein telescope [3–11] will provide a solid answer on whether the inflationary era [12–15] took place, or at least in the less optimistic scenario will further constrain it. Although the stage 4 CMB experiments will directly probe the B-mode seeds of inflation in the CMB temperature polarization pattern, the future gravitational waves will probe the inflationary modes (that is, the tensor modes generated by the curvature perturbations during inflation) that reentered the Hubble horizon shortly after the inflationary era during the reheating and the radiation domination era. These two eras are hypothetical stages of our Universe's evolution, which are quite mysterious themselves. During the radiation era it is theorized that many hypothetical phenomena like the electroweak symmetry breaking have taken place. Regarding the latter, it provides an elegant solution to the observed baryon asymmetry in the Universe since, in many scenarios, the electroweak symmetry breaking stage

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is followed by a strong first order phase transition, which is an era of nonthermal equilibrium, according to the Sakharov criteria [16]. Apart from the baryon asymmetry, the electroweak first order phase transition can in principle produce primordial gravitational waves and these can be detected by future interferometers [11,17–40]. These first order phase transitions are generated and further assisted by singlet extensions of the Standard Model coupled only to the Higgs particle [41–58] or can be generated by Higgs self-couplings materialized by higher dimensional nonrenormalizable operators [22,25,40].

Modified gravity in its various forms [59–63] may consistently describe the acceleration eras of our Universe, that is the inflationary and the dark energy eras, and possibly all the eras in between, without the need of scalar fields, however, regarding the dark matter problem that still persists, one has to be inceptive for its consistent description. Weakly interacting massive particles (WIMPs) still seem to be undetected, or well hidden in a Standard Model hidden sector; thus currently the focus and interest is on light mass particles, such as the axion [64–96], see also [97,98] for reviews and also an interesting simulation [99] for μeV range axions. Recently, some experimental proposals are further proposed for finding the axion [100], and also motivation for axions having mass of the order $m_a \sim \mathcal{O}(10^{-10})$ eV is provided by recent gamma ray bursts observations [101,102]. In this article we shall consider the effects of a direct nonrenormalizable coupling between the axion and the Higgs on the energy density of the primordial gravitational waves. Couplings between axions and Higgs have been considered in the literature [103–105] in a different context; however, in this paper we shall assume that the couplings are in terms of higher order nonrenormalizable operators. These higher order nonrenormalizable operators originate from a scale M way higher than the electroweak scale, a fact that is motivated by the lack of new particle observations in the Large Hadron Collider (LHC) beyond the electroweak breaking scale [22]. When the electroweak symmetry breaking occurs, nearly at a temperature $T \sim 100$ GeV [37], these higher order operators modify the axion potential and cause a new minimum to it. Eventually, the axion oscillations at the origin of the axion potential are destabilized after some time because these are unbounded, and the axion is allowed to swiftly roll down to the new potential minimum. Since the axion rolls swiftly in its new potential minimum, it experiences a kination era and its energy density scales as $\rho_a \sim a^{-6}$. With regard to the kination era and its relation with light axion relics see for example [106–109]. This short kination era may modify somewhat the background equation of state (EOS) parameter from the radiation domination value $w = 1/3$ to a deformed value closer to the kination era value $w = 1$, depending on whether the axion composes all the dark matter or some portion of it. Eventually when the axion reaches its new minimum, the latter is energetically unfavorable compared to the Higgs vacuum, and thus the axion minimum decays to the Higgs minimum. Hence the actual vacuum of the Universe is the Higgs minimum and the axion returns to the origin of its potential. Quantum fluctuations generate the axion oscillations at the origin since the dominant potential term for small field values is $\sim\phi^2$, thus the axion starts to redshift again as dark matter. After some considerable amount of time, the axion is again destabilized due to the unbounded motion and starts to swiftly roll its potential minimum again, and a new instant kination era occurs for it. Thus the background EOS is changed again slightly, and the procedure described is continued perpetually. It is thus possible that during the postelectroweak breaking radiation domination era, the Universe might have one or more deformations of its background EOS $w = 1/3$ to some value closer to the kination era value $w = 1$. We shall assume in a conservative way that the background EOS value during the kinetic phases of the axion is $w = 0.5$, and we shall investigate the effects of these axion kinetic deformations on the energy spectrum of the primordial gravitational waves. As we shall see, the effects can be measurable in some scenarios, depending on the value of the total background EOS parameter, the total number of these short kination eras, and finally on which inflationary scenario materializes the inflationary era. Also we briefly discuss the effects of the axion movement towards its new minimum in the case that the axion initially slow rolls towards it, and also we investigate the effects of high temperature on the scenarios we discussed above. This study however is purely academic, since the axion never thermalizes with its background, thus the high temperature effects are not justified. We included however the high temperature effects just out of curiosity and for academic interest.

II. AXION AND THE HIGGS PORTAL VIA DIMENSION SIX OPERATORS AND DIMENSION EIGHT OPERATORS

The axion scalar is the prominent dark matter candidate currently. Its mass is likely in the sub-eV region, perhaps quite small, ranging from 10^{-6}–10^{-27} eV, according to different studies [67]. In this paper we shall consider the axion evolution in the postinflationary era, well inside the reheating era and specifically during the electroweak breaking regime, which occurs when the temperature of the Universe drops to $T \sim 100$ GeV, after the Universe reached its high reheating temperature. Regarding the reheating temperatures, it is still questionable whether it was too high, but for the sake of the argument we shall assume that the reheating temperature was in the range 10^4–10^7 GeV, although there are studies that predict a much lower reheating temperature. Also we shall assume that the axion mass is of the order $m_a \sim \mathcal{O}(10^{-10})$ eV motivated by studies predicting such a mass based on gamma ray burst observations [101,102].

We shall mainly be interested in the misalignment axion scenario [67,71], in the context of which the primordial Peccei-Quinn $U(1)$ symmetry is broken during inflation and the axion is misaligned from the vacuum state having a quite large initial value $\phi_i \sim f_a$, where f_a is the axion decay constant, which will be assumed to be larger than $f_a > 10^9$ GeV. The original axion potential has the form

$$V_a(\phi) = m_a^2 f_a^2 \left(1 - \cos\left(\frac{\phi}{f_a}\right)\right). \tag{1}$$

The term $\frac{\phi}{f_a}$ can quantify very well the misalignment values of the axion, so when $\phi/f_a < 1$, the axion misalignment potential that describes the misalignment axion from the minimum of the potential is approximated as follows:

$$V_a(\phi) \simeq \frac{1}{2} m_a^2 \phi^2, \tag{2}$$

an approximation that is valid when $\phi < f_a$ and is expected to no longer hold true when $\phi \sim f_a$. This will be somewhat important at a later stage of our analysis. Regarding the initial kinetic energy of the axion, there are two mainstream scenarios that we shall take into account: the ordinary misalignment axion scenario, according to which the axion starts with zero kinetic energy and rolls slowly to the minimum of the potential [67], and the second scenario is the kinetic axion case [71]. According to the latter scenario, the axion starts with a large kinetic energy. In both cases, when the Hubble rate of the Universe becomes of the order of the axion mass $H \sim m_a$, the axion commences rapid oscillations and redshifts as dark matter. However, this era of oscillations is significantly delayed in the kinetic axion case, because an era of kination follows after the quasi–de Sitter inflationary era, and thus the axion has enough kinetic energy to climb uphill to its potential, and then it starts to roll towards the minimum of its potential. In the kinetic misalignment axion case, the reheating temperature is presumably lower, compared to the ordinary misalignment axion case, because the inflationary era is prolonged by several e-foldings in the kinetic axion case [93,94]. The two scenarios are described pictorially in Fig. 1. We shall be interested in the effects of the electroweak symmetry breaking on the axion evolution and the implications on the primordial tensor modes crossing the horizon around that era. To this end, we shall assume that some higher dimensional operators of the axion are extremely weakly coupled to the Higgs sector solely, while no other connection exists between the axion and the Higgs sector. Thus we shall investigate the axion's evolution effects on primordial gravitational waves through the Higgs portal during the electroweak breaking epoch that is presumed to occur at $T \sim 100$ GeV [36,37]. From the reheating

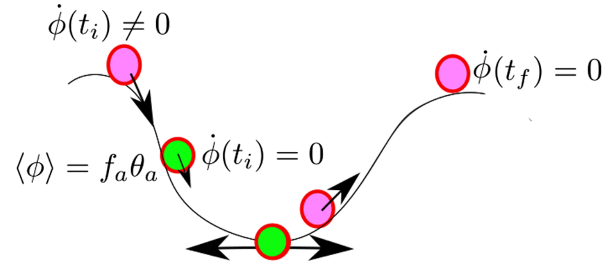

FIG. 1. The kinetic and ordinary misalignment axion models. In the kinetic axion case, the axion has a large kinetic energy, and the oscillations era is delayed.

temperature down to 100 GeV the Universe proceeds without any changes with regard to its vacuum structure. The Higgs portal will be constituted by six- and eight-dimensional operators active at a scale M, which is assumed to be in the range $M \sim 20$–100 TeV. Essentially, what we have is an extremely weakly coupled theory originating at multiple TeVs scale. The proposed Higgs-axion potential at tree order is the following:

$$V(\phi, h) = V_a(\phi) - m_H^2 |H|^2 + \lambda_H |H|^4 - \lambda \frac{|H|^2 \phi^4}{M^2} + g \frac{|H|^2 \phi^6}{M^4}, \tag{3}$$

with $V_a(\phi)$ being defined in Eq. (1), the Higgs field prior to electroweak symmetry breaking is $H = \frac{h + i h_1}{\sqrt{2}}$ being the Higgs scalar, $m_H = 125$ GeV [110] is the Higgs boson mass, and λ_H is the Higgs self-coupling, which are related as $\frac{v}{\sqrt{2}} = (\frac{-m_H^2}{\lambda_H})^{\frac{1}{2}}$, where v is the electroweak symmetry breaking scale $v \simeq 246$ GeV. Furthermore, m_a is the axion mass which will be a free parameter in our theory, M is the high scale of the effective theory in which the nonrenormalizable dimension six and dimensions eight operator originate from. These dimension six and dimension eight higher order nonrenormalizable operators originate from an effective theory active at the scale M, which will be assumed to be way higher than the electroweak scale, of the order $M = 20$–100 TeV, a fact that is further motivated by the lack of new particle observations in the LHC beyond the electroweak breaking scale [22]. Also, λ and g are the dimensionless couplings or the Wilson coefficients of the higher order effective theory of the axion to the Higgs, which will be assumed to be small of the order $\lambda \sim \mathcal{O}(10^{-20})$ and $g \sim \mathcal{O}(10^{-5})$. In this paper we shall assume that the electroweak symmetry breaking occurs and that a first order phase transition occurs in the electroweak sector. This can be achieved in various ways, for example via some scalar extension of the Higgs sector, only coupled to the Higgs sector [41–58], or by higher dimensional operators of Higgs self-couplings [22,25,40]. The couplings of the Higgs and axion particle ensure that the axion cannot affect

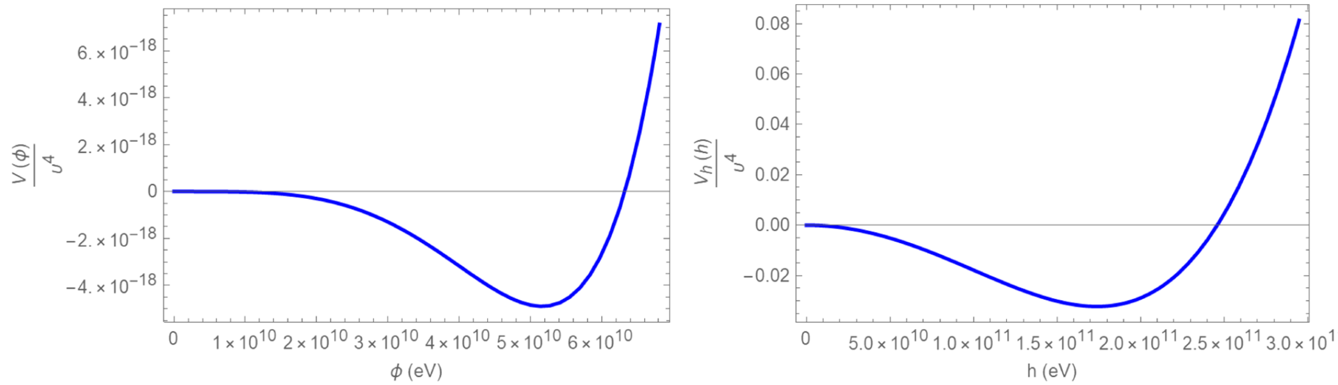

FIG. 2. The axion effective potential for $m_a \sim 10^{-10}$ eV, $M = 20$ TeV, $\lambda \sim \mathcal{O}(10^{-20})$, and $g \sim \mathcal{O}(10^{-5})$ (left plot) and the Higgs potential after electroweak symmetry breaking (right plot). The Higgs potential is much deeper in its minimum value compared to the axion potential. The axion vacuum is unstable compared to the Higgs vacuum.

the electroweak sector, even after the electroweak breaking. Before the breaking, the effects are well hidden due to the weakness of the interaction. However, as we will see, the electroweak breaking can affect the axion sector, causing the axion to evolve differently after it occurs. Indeed, after the electroweak symmetry breaking occurs at $T \sim 100$ GeV, the Higgs field acquires a vacuum expectation value, thus $H = v + \frac{h+ih_1}{\sqrt{2}}$, therefore the axion potential becomes modified, since the higher dimensional operators induce self-interaction terms to the axion potential. Hence, the resulting modified axion effective potential $\mathcal{V}_a(\phi)$ after electroweak breaking is

$$\mathcal{V}_a(\phi) = V_a(\phi) - \lambda \frac{v^2\phi^4}{M^2} + g\frac{v^2\phi^6}{M^6}, \tag{4}$$

with $V_a(\phi)$ being defined in Eq. (1). These electroweak symmetry breaking induced self-couplings to the axion cause vacuum instability of the axion sector. In order to pictorially see this vacuum instability in the axion sector let us use the numerical values $m_a \sim 10^{-10}$ eV, $M = 20$ TeV, and we take Wilson coefficients of the higher dimensional operators to be of the order $\lambda \sim \mathcal{O}(10^{-20})$ and $g \sim \mathcal{O}(10^{-5})$. In Fig. 2 we plot the axion effective potential (left plot) and the Higgs effective potential (right plot). As it can be seen the total effective potential in the Higgs minimum direction $(h, \phi) = (v, 0)$ is much deeper compared to the axion minimum $(h, \phi) = (0, v_s)$. It is apparent that the dimension six and dimension eight operators render the axion sector unstable, since the axion is now allowed to acquire a vacuum expectation value. This could be catastrophic if the axion is the only component of dark matter, or is some considerable portion of the dark matter in the Universe. But this is not so as we will see in the next section. Although the Universe can have two physical vacua that can dominate, the axion vacuum $(h, \phi) = (0, v_s)$ and the Higgs vacuum $(h, \phi) = (v, 0)$, the axion vacuum is unstable, and immediately decays to the much deeper Higgs vacuum. We will discuss the physical picture of the axion instability and its implications for primordial gravitational waves in the next two subsections.

We need to note that even including the one-loop correction in the axion effective potential,

$$V^{1-\text{loop}}(\phi) = \frac{m_{\text{eff}}^4(\phi)}{64\pi^2}\left(\ln\left(\frac{m_{\text{eff}}^2(\phi)}{\mu^2}\right) - \frac{3}{2}\right), \tag{5}$$

does not change the overall physical picture, where $m_{\text{eff}}^2(\phi)$ is

$$m_{\text{eff}}^2(\phi) = \frac{\partial^2 V(\phi, h)}{\partial\phi^2} = m_a^2 - \frac{6\lambda v^2\phi^2}{M^2} + \frac{15 g v^2\phi^4}{M^4}, \tag{6}$$

and μ denotes the renormalization scale. We have checked this one-loop contribution for various values of the Wilson coefficients for $M \sim 20$–100 TeV and for values of the renormalization scale μ. The only issue is that the one-loop contribution may become complex for some field ϕ values and for some values of the Wilson coefficients, which also indicates vacuum instability. However, if temperature corrections were included, this imaginary part would cancel. But the axion never thermalizes with the particle environment, so one does not have to worry about high temperature corrections. In a later section though, just for academic curiosity, we shall include high temperature corrections to see the effects in the physical picture. The result is the same although high temperature delays the physics we shall describe.

A. Metastable vacuum decay for both scenarios and physical description of the resulting phenomenology

As we have seen in the previous subsection, after the electroweak symmetry breaking in the Higgs sector, the dimension six and dimension eight operators render the axion effective potential unstable at the origin and it

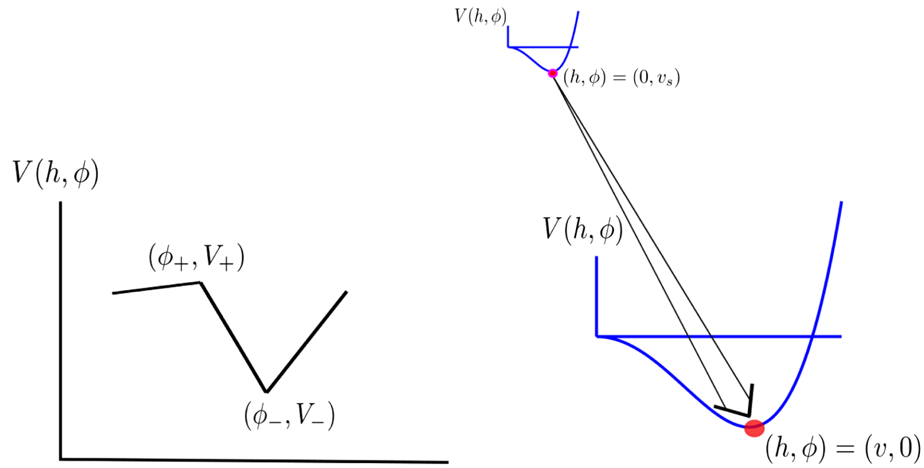

FIG. 3. Simplified triangular approximation for vacuum decay in the axion-Higgs effective potential (left plot) and the vacuum decay between the axion vacuum to the Higgs electroweak vacuum (right plot).

may develop a vacuum expectation value. Thus after electroweak breaking, the Universe has two competing vacua, the one corresponding to the Higgs field $(h,\phi) = (v,0)$ and the one corresponding to the axion field $(h,\phi) = (0,v_s)$. Which vacuum will dominate and which physics will be materialized depends on which is energetically favorable. Depending on the values of the total axion-Higgs potential in the two distinct vacua, the one vacuum may be highly unstable and can tunnel quantum mechanically to the other vacuum according to the Coleman description. The condition that determines which vacuum is energetically favorable compares the depth of the effective potential for the two distinct vacua, so in our case we have

$$V(\phi,h)|_{(h,\phi)=(0,v_s)} \gg V(\phi,h)|_{(h,\phi)=(v,0)}, \tag{7}$$

which essentially translates to the simple statement that the "deeper" vacuum energetically dominates over the other one, so basically the energetically unfavorable vacuum is unstable and decays to the stable vacuum. In our case, as can be seen in Fig. 2, the Higgs vacuum is by far more energetically favorable compared to the axion vacuum, thus the latter is highly unstable so it will decay eventually to the Higgs vacuum. In order to quantify this picture, let consider the triangular approximation used in Ref. [111], so pictorially the physical situation is described in Fig. 3, a simplified form of which is represented in the left plot. In the triangular approximation, let ϕ_- denote the Higgs minimum $(h,\phi) = (v,0)$ of the axion Higgs effective potential, V_- the value of the effective potential at the Higgs minimum $(h,\phi) = (v,0)$, and V_+ and ϕ_+ the corresponding quantities for the axion minimum $(h,\phi) = (0,v_s)$. Since in our case, the barrier connecting the two vacua in the triangular approximation is practically elevated to the same level as the axion minimum, the tunneling rate parameter B is

$$B = \frac{2\pi}{3}\frac{\Delta\phi^4\tilde{\lambda}}{\Delta V}, \tag{8}$$

where in our case $\Delta\phi = \phi_+ - \phi_- \simeq \phi_-$, $\Delta V = V_+ - V_- \simeq V_-$, and $\tilde{\lambda}$, which is the steepness of the Higgs vacuum, is approximately of the order unity $\tilde{\lambda} \sim \mathcal{O}(1)$. Thus the tunneling rate parameter B, which enters in the tunneling rate $\Gamma \sim e^{-B}$ of the axion vacuum $(h,\phi) = (0,v_s)$ to the Higgs vacuum $(h,\phi) = (v,0)$, is a very small parameter of the order $B \sim \mathcal{O}(10^{-42})$, thus the tunneling rate is maximal and the probability of tunneling is of the order of unity. Hence the axion vacuum is indeed highly metastable in our case and instantly decays to the Higgs vacuum in the way described in Fig. 3. This is because the decay probability of a metastable vacuum per unit time and volume γ is proportional to $\gamma \sim e^{-B}$. After a time average $T \sim 1/\gamma$, the metastable vacuum decays to the stable vacuum. Since in our case, γ takes the maximum value, the time average is minimal and thus the axion vacuum instantly decays to the Higgs vacuum.

1. Metastable vacuum decay and the evolution of the axion field

We discussed the development of the axion minimum in the axion-Higgs effective potential, we showed that the axion minimum is unstable and it instantly decays to the Higgs minimum, so now let us qualitatively discuss how the axion phenomenology is affected by the metastable axion minimum and how does this effect might affect the overall evolution of the Universe to some large or small extent.

First, exactly at the electroweak breaking epoch, the axion potential develops a new minimum. Thus the axion, which prior to electroweak breaking was performing oscillations around the origin, now starts to feel the second minimum of the potential. The oscillations of the axion at the origin are unstable, and thus the origin becomes an unstable stationary point. Hence, eventually after some time, which depends on the amplitude of the oscillations and the initial conditions at the electroweak breaking scale, the axion will start to roll down to its new potential minimum, and its large potential energy—for the axion standards—is converted to kinetic energy swiftly. It is thus obvious that since the axion becomes effectively a kination fluid for some small period of time, with that period of time being the duration of the motion of the axion towards its new minimum. Hence for this short period, the axion energy density scales as $\rho_a \sim a^{-6}$, and by considering the fact that this occurs after the electroweak transition, so basically deeply in the radiation domination era, this effect might disturb significantly the total background EOS of the Universe during the radiation domination era, changing it from the value $w = 1/3$ to some different larger value, closer to the kination limit $w = 1$, for example $1/3 < w < 0.6$. When the axion reaches its new potential minimum, the new axion vacuum decays instantly to the Higgs vacuum and the axion returns to the origin of its effective potential. Quantum fluctuations then will generate small oscillations about the origin, since the axion potential for small ϕ values is $\mathcal{V}_a(\phi) \sim \frac{1}{2} m_a^2 \phi^2$, thus the axion scales as dark matter $\rho \sim a^{-3}$ for the period that the oscillations occur. When the oscillations amplitude grows larger, the axion starts again to roll swiftly to the metastable axion potential minimum, a new kination evolution for the axion occurs, and the background EOS can be disrupted again during the radiation domination era. Once the axion reaches swiftly the new minimum, the metastable minimum decays and the axion again returns to the origin. Thus this scenario may proceed until the radiation domination era ends, and the Universe enters to the matter domination era, and may continue even up to present day. This behavior is pictorially described in Fig. 4. Therefore, in the case at hand, the axion energy density still scales as cold dark matter $\rho_a \sim a^{-3}$ when the axion is oscillating in the origin, but the evolution is swiftly interrupted by very short kination eras. The latter last for a very small amount of time until the new axion minimum is reached. Then the axion returns to the origin after the axion vacuum decays to the Higgs vacuum. In the next section we shall consider the effects of these short kination eras and how these may affect the primordial gravitational waves. We need to note that only the modes that reenter the Hubble horizon during the radiation domination era after the electroweak breaking will be affected. This fact however strongly depends on how large the reheating temperature was, thus the gravitational wave implications of these short axion kination eras, which will disturb the radiation domination era, should be considered in detail. This is considered in the next subsection.

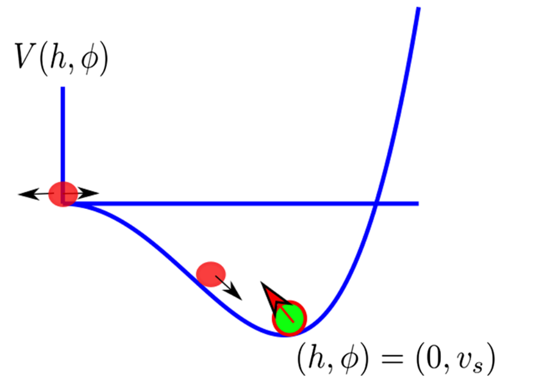

FIG. 4. The axion potential $V(\phi)$ after the electroweak symmetry breaking develops a new minimum at the direction $(h, \phi) = (0, v_s)$. The axion oscillations at the origin are unbounded and the axion starts swiftly to roll down its potential to the new minimum. The axion undergoes a swift kination era for which all the potential energy is transformed to kinetic energy. Once it reaches the new minimum, the minimum decays to the energetically favorable Higgs minimum $(h, \phi) = (v, 0)$, and the kinetic energy and the decay energy is transferred to axion that returns to the origin. Quantum fluctuations generate again axion oscillations at the origin, which after some considerable time become unbounded and the same kination procedure is repeated perpetually.

B. Implications on primordial gravitational waves

In this section we shall discuss the general consequences of the electroweak symmetry breaking on gravitational waves via the axion sector. As we demonstrated in the previous section, the electroweak symmetry breaking induces a new vacuum state for the axion scalar field, to which the axion rolls rapidly once it gets destabilized from the local maximum at the origin of its potential. This fast roll may have measurable consequences for the background total EOS of the Universe, since the axion solely experiences a kination era and its energy density scales as $\rho_a \sim a^{-6}$, thus it evolves slower than radiation. Thus if the axion composes one part or all dark matter, then this can affect the total EOS parameter w, changing it from the standard radiation value $w = 1/3$ to some alternative higher value closer to the value $w = 1$, which would indicate a kination era. If the axion is the sole dark matter component, then the total EOS parameter would be very close to the kination value, but for demonstration purposes we shall assume that the total EOS parameter changes to $w = 0.5$. A larger value than this proves to have more dramatic effects on the detection of primordial gravitational waves, but we shall take a moderate path. Now regarding the axion rolling eras, these occur in a rapid way, and once the axion reaches the new minimum, the new axion vacuum decays rapidly to the Higgs vacuum. Thus the axion returns to the origin of its potential to its initial vacuum state, and the Universe is described by the vacuum $(h, \phi) = (v, 0)$. However, due to quantum fluctuations, the axion oscillations at the origin, generated by the dominant part of the potential at the origin $\sim\phi^2$, are destabilized again after some considerable amount of time and the axion starts to roll again to the potential minimum, since the other terms of the potential that cause the new minimum are starting to dominate. The kination era occurs again for the axion. This behavior continues perpetually for the axion, with the latter experiencing instant kination eras during its postelectroweak breaking evolution. This kind of behavior can affect the last stages of the radiation domination era, relevant for LISA, BBO, and DECIGO physics, depending on how low the reheating temperature was. In principle, no one can know how large the reheating temperature was, there is motivation that it can be as low as some hundreds MeV [112]. This in principle could put in peril the electroweak phase transition; however for the sake of the argument, we shall assume that the reheating temperature can be either $T_R = 10^7$ GeV or $T_R = 10^4$ GeV. The case of a lower reheating temperature could be realized if for example a kinetic axion is controlling the postinflationary evolution, while larger reheating temperatures can be realized by the canonical misalignment axion model (see previous sections on this). The electroweak phase transition is theorized to have occurred at approximately $T \sim 100$ GeV, and if assisted by scalar fields or by other mechanisms, it can be first order. This could also generate another source of primordial gravitational waves for this era which can work in a synergistic way with the amplification of gravity waves due to the effects we shall describe in this section, but for simplicity we omit the primordial gravitational waves originating by the electroweak phase transition. For a reheating temperature as high as $T_R = 10^7$ GeV, the tensor modes reentering the horizon at the reheating temperature have a wave number of the order $k_R = 1.19 \times 10^{15}$ Mpc^{-1}. Hence at the electroweak phase transition the relevant wave numbers would be of the order $k_h \sim 10^{10}$ Mpc^{-1}. Accordingly, for reheating temperature as high as $T_R = 10^4$ GeV, the tensor modes reentering the horizon at the reheating temperature have a wave number of the order $k_R = 1.19 \times 10^{12}$ Mpc^{-1}. Hence, in this case at the electroweak phase transition the relevant wave numbers would be of the order $k_h \sim 10^9$ Mpc^{-1} approximately. For academic purposes and just from curiosity, we shall consider three scenarios: first that during the radiation domination era there is only axion kinetic epoch, second, that there are two kination epochs, and third three kination epochs. For the high reheating epochs we shall assume that the kination epoch modes are for example $k_{t_1} = 6.5 \times 10^{10}$ Mpc^{-1}, $k_{t_1} = 10 \times 10^{10}$ Mpc^{-1}, and $k_{t_1} = 15 \times 10^{10}$ Mpc^{-1} and, accordingly for the low reheating temperature, $k_{t_1} = 6.5 \times 10^9$ Mpc^{-1}, $k_{t_1} = 10 \times 10^9$ Mpc^{-1}, and $k_{t_1} = 15 \times 10^9$ Mpc^{-1}. Now, if the total EOS parameter of the background is changed to w at some relevant wave number k_μ, the h^2-scaled energy spectrum of the primordial gravitational waves changes by a multiplication factor $\sim (\frac{k}{k_\mu})^{r_c}$, where $r_c = -2(\frac{1-3w}{1+3w})$ [113]. Thus for three distinct such deformations of the radiation domination era with $w = 0.5$, one should include three such multiplication factors to the h^2-scaled energy spectrum of the primordial gravitational waves. We shall denote this deformation of the kination eras as $S_k(f)$, and the total h^2-scaled energy spectrum of the primordial gravitational waves is [114–138] (see also Ref. [139] for a recent review),

$$\Omega_{\mathrm{gw}}(f) = S_k(f) \times \frac{k^2}{12H_0^2} r\mathcal{P}_\zeta(k_{\mathrm{ref}}) \left(\frac{k}{k_{\mathrm{ref}}}\right)^{n_T} \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^2 \times \left(\frac{g_*(T_{\mathrm{in}})}{g_{*0}}\right) \left(\frac{g_{*s0}}{g_{*s}(T_{\mathrm{in}})}\right)^{4/3} \left(\frac{\overline{3j_1(k\tau_0)}}{k\tau_0}\right)^2 \times T_1^2(x_{\mathrm{eq}}) T_2^2(x_R),$$

where $k_{\mathrm{ref}} = 0.002$ Mpc^{-1} is the CMB pivot scale and $S_k(f)$ is the contribution coming from the axion abrupt kination eras. As it can be seen in the energy spectrum above, the contributions from the inflationary era are also included. With regard to the inflationary era, we shall consider two models that can describe it: the R^2 model, which yields a negative tensor spectral index, and an Einstein-Gauss-Bonnet theory, which yields a blue-tilted tensor spectrum. Specifically the R^2 model yields a tensor spectral index $n_T = -0.000375$ and $r = 0.003$, and the Einstein-Gauss-Bonnet theory we shall consider is compatible with the GW170817 even yielding a gravitational wave speed equal to that of light's in vacuum, with the corresponding observational indices being $n_T = 0.37$ and $r = 0.02$ [135,140,141]. Let us briefly recall the theoretical frameworks of R^2 and Einstein-Gauss-Bonnet gravity here, which yield the aforementioned values for the inflationary observational indices. Regarding the vacuum $F(R)$ gravity in the presence of a kinetic axion, which does not affect at all the inflationary era, the action is

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}F(R) - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + \mathcal{L}_m\right], \tag{9}$$

but recall that, as it was shown in [94], the axion does not affect the inflationary era, just the duration of it. Thus, the theory that mainly controls the dynamics of inflation is the $F(R)$ gravity part:

$$F(R) \simeq R + \frac{1}{M^2}R^2. \tag{10}$$

Accordingly, the field equations become

$$\ddot{H} - \frac{\dot{H}^2}{2H} + \frac{HM^2}{2} = -3H\dot{H}. \tag{11}$$

And, due to the slow-roll assumptions,

$$-\frac{M^2}{6} = \dot{H}, \tag{12}$$

we get

$$H(t) = H_I - \frac{M^2}{6}t. \tag{13}$$

The cosmological perturbations are quantified in a perturbative way by the "slow-roll" indices [59,135],

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \qquad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}, \qquad \epsilon_3 = \frac{\dot{F}_R}{2HF_R}, \qquad \epsilon_4 = \frac{\dot{E}}{2HE}, \tag{14}$$

with E for the case at hand reads

$$E = F_R + \frac{3\dot{F}_R^2}{2\kappa^2\dot{\phi}^2}. \tag{15}$$

Following closely Ref. [94], to which we refer the reader for more details, by also taking into account the kinetic axion at the end of inflation, the spectral index of tensor and scalar perturbations at the end of inflation and the tensor-to-scalar ratio take the form, $n_s \sim 1 - \frac{2}{N}$, $n_T \sim -\frac{1}{N^2}$, and $r \sim \frac{12}{N^2}$, where N is the e-foldings number. So for a kinetic axion theory N must be larger than 60 e-foldings, and we took 65 for our calculations. Now regarding the Einstein-Gauss-Bonnet theory, the action is [135,140,141]

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{2}\xi(\phi)\mathcal{G}\right), \tag{16}$$

where R denotes the Ricci scalar, $\kappa = \frac{1}{M_p}$ where M_p is the reduced Planck mass. Also $\mathcal{G}$ stands for the Gauss-Bonnet invariant which is $\mathcal{G} = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ and $R_{\alpha\beta}$ and $R_{\alpha\beta\gamma\delta}$ denote the Ricci and Riemann tensor, respectively.

Following the argument of [135,140,141] regarding the gravitational wave speed being equal to unity, the slow-roll indices of inflation are simplified, as follows [140]

$$\epsilon_1 \simeq \frac{\kappa^2}{2}\left(\frac{\xi'}{\xi''}\right)^2, \tag{17}$$

$$\epsilon_2 \simeq 1 - \epsilon_1 - \frac{\xi'\xi'''}{\xi''^2}, \tag{18}$$

$$\epsilon_3 = 0, \tag{19}$$

$$\epsilon_4 \simeq \frac{\xi'}{2\xi''}\frac{\mathcal{E}'}{\mathcal{E}}, \tag{20}$$

$$\epsilon_5 \simeq -\frac{\epsilon_1}{\lambda}, \tag{21}$$

$$\epsilon_6 \simeq \epsilon_5(1 - \epsilon_1), \tag{22}$$

where $\mathcal{E} = \mathcal{E}(\phi)$ and $\lambda = \lambda(\phi)$ are defined as follows:

$$\mathcal{E}(\phi) = \frac{1}{\kappa^2}\left(1 + 72\frac{\epsilon_1^2}{\lambda^2}\right), \qquad \lambda(\phi) = \frac{3}{4\xi''\kappa^2 V}. \tag{23}$$

And hence the observational indices of inflation are defined as

$$n_{\mathcal{S}} = 1 - 4\epsilon_1 - 2\epsilon_2 - 2\epsilon_4, \tag{24}$$

$$n_{\mathcal{T}} = -2(\epsilon_1 + \epsilon_6), \tag{25}$$

$$r = 16\left|\left(\frac{\kappa^2 Q_e}{4H} - \epsilon_1\right)\frac{2c_A^3}{2 + \kappa^2 Q_b}\right|, \tag{26}$$

where c_A is the sound speed,

$$c_A^2 = 1 + \frac{Q_a Q_e}{3Q_a^2 + \dot{\phi}^2(\frac{2}{\kappa^2} + Q_b)}, \tag{27}$$

and with

$$Q_a = -4\dot{\xi}H^2, \qquad Q_b = -8\dot{\xi}H, \qquad Q_t = F + \frac{Q_b}{2},$$
$$Q_c = 0, \qquad Q_e = -16\dot{\xi}\dot{H}, \tag{28}$$

can be further simplified as follows:

$$r \simeq 16\epsilon_1, \tag{29}$$

$$n_{\mathcal{T}} \simeq -2\epsilon_1\left(1 - \frac{1}{\lambda} + \frac{\epsilon_1}{\lambda}\right). \tag{30}$$

Using the following model,

$$\xi(\phi) = \beta\exp\left(\left(\frac{\phi}{M}\right)^2\right), \tag{31}$$

where β is a dimensionless parameter, and M denotes a free parameter with mass dimensions $[m]^1$. For this model, the observational indices become

$$n_{\mathcal{S}} \simeq -1 - \frac{\kappa^2 M^4\phi^2}{(M^2 + 2\phi^2)^2} + \frac{4\phi^2(3M^2 + 2\phi^2)}{(M^2 + 2\phi^2)^2} + \frac{4608\beta^2\phi^6 e^{\frac{2\phi^2}{M^2}}(6\gamma\phi^2 + 16\beta e^{\frac{\phi^2}{M^2}}(M^2 + \phi^2) + 9\gamma M^2)}{(M^2 + 2\phi^2)^4(3\gamma + 4\beta e^{\frac{\phi^2}{M^2}})^3} \tag{32}$$

and

$$n_{\mathcal{T}} \simeq \frac{\phi^2(-4\beta e^{\frac{\phi^2}{M^2}}(M^4(3\kappa^2\phi^2-2)+\kappa^2M^6-8M^2\phi^2-8\phi^4)-3\gamma\kappa^2M^4(M^2+2\phi^2))}{(M^2+2\phi^2)^3(3\gamma+4\beta e^{\frac{\phi^2}{M^2}})}, \tag{33}$$

while the tensor-to-scalar ratio reads

$$r \simeq \frac{8\kappa^2M^4\phi^2}{(M^2+2\phi^2)^2}. \tag{34}$$

For $\mu = [22.09147657871, 22.09147657877]$, $\beta = -1.5$, $\gamma = 2$, and for $N = 60$ e-foldings we obtain $n_{\mathcal{T}} = [0.378856, 0.379088]$, and we will use this model for the primordial gravitational waves predictions of this theory. Now let us confront the corresponding theories with the sensitivity curves coming from all future and present (NANOGrav) gravitational wave experiments, taking also into account the abrupt short kination eras of the axion sector, with one, two, and three kinetic eras occurring during the radiation domination era. In our analysis we shall include the latest constraints coming from the LIGO/VIRGO collaboration [142] that indicate that the energy spectrum of the primordial gravitational waves must be $\Omega_{GW} \leq 5.8\times10^{-9}$ for a flat and frequency independent background for frequencies in the range (20–76.6) Hz (LIGO VIRGO BOUND I), and furthermore $\Omega_{GW} \leq 3.4\times10^{-9}$ for a spectral index of $2/3$ power-law background in the range (20–90.6) Hz (LIGO VIRGO BOUND II), and $\Omega_{GW} \leq 3.9\times10^{-10}$ when the spectral index is 3, in the range (20–291.6) Hz (LIGO VIRGO BOUND III). Moreover, the BBN bound is included [143]. In Figs. 5 and 6 we present the R^2 model results with one, two, and three kination peaks, while in Figs. 7 and 8 we present the Einstein-Gauss-Bonnet model results with again one, two, and three kinetic peaks, for two distinct reheating temperatures, namely $T_R = 10^7$ GeV and $T_R = 10^4$ GeV. Now let us discuss the resulting phenomenology and we start with the R^2 model. For a small reheating temperature, the primordial gravitational wave energy spectrum remains undetectable, however for a large reheating temperature with three and two kination peaks can be detected by the BBO and DECIGO experiments. In the Einstein-Gauss-Bonnet case, the primordial gravitational waves are detectable even for a reheating temperature as low as $T_R = 10^4$ GeV and can be detected from LISA, DECIGO, and BBO, even for one kination peak. In fact for a large reheating temperature, the two and three kination peaks are already excluded by the LIGO/VIRGO constraints, as can be seen in Fig. 7. We should note that we did not include in our study the synergistic effect of the electroweak phase transition, which if it is of first order, this could affect the energy spectrum when the temperature of the Universe is 100 GeV and could further amplify the spectrum. We solely considered the effects of the axion kination eras, for simplicity, in order to discriminate the effects of the kination eras. Before closing, we stress the effect of the background EOS parameter w that we assume is equal to $w = 0.5$. If this is closer to the kination value, for example $w = 0.9$, then the effects on the primordial gravitational

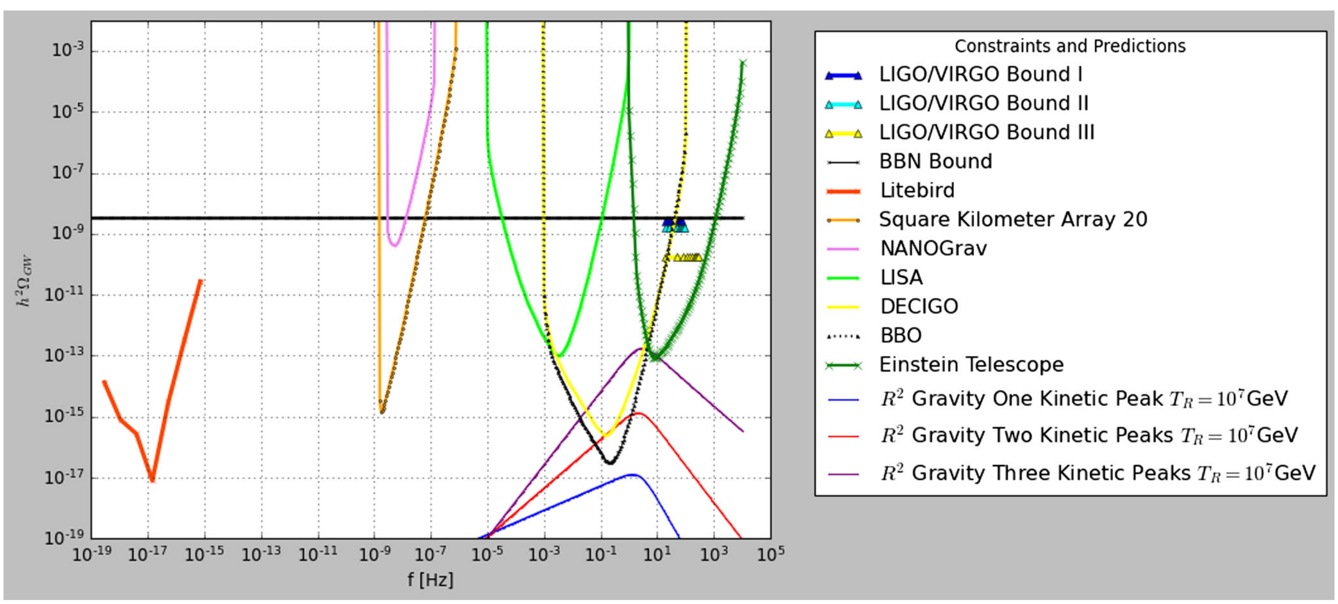

FIG. 5. The h^2-scaled gravitational wave energy spectrum for the R^2-inflation canonical misalignment axion model, confronted with the sensitivity curves of gravitational waves experiments for the reheating temperature being high $T_R = 10^7$ GeV.

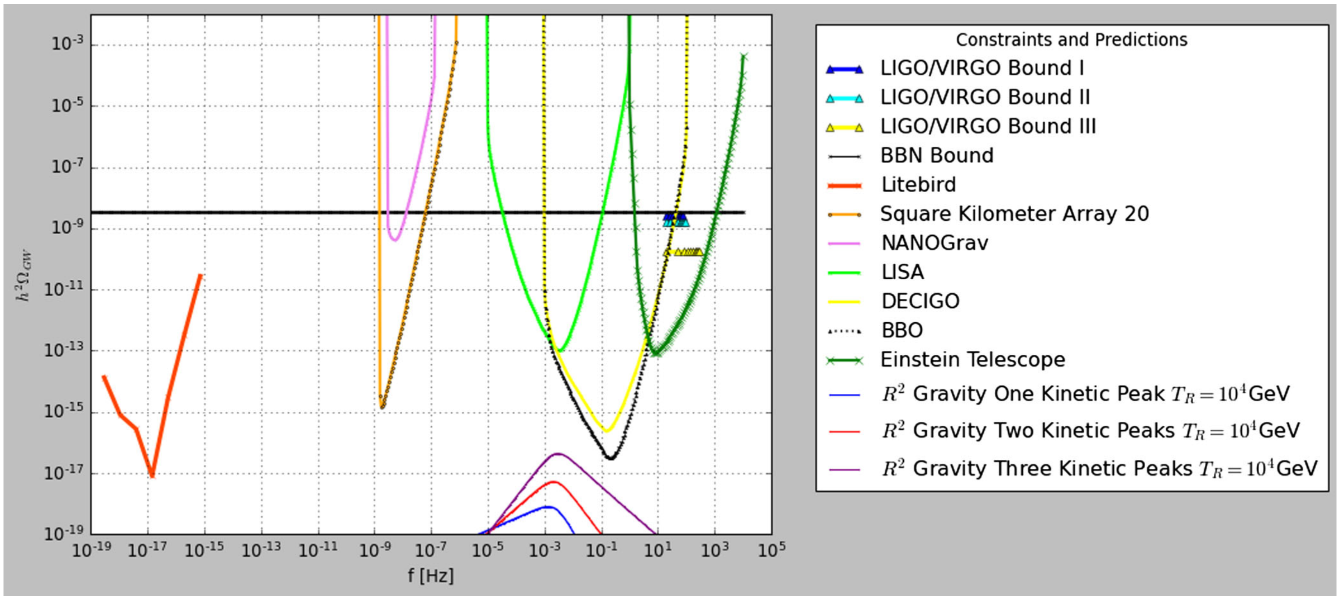

FIG. 6. The h^2-scaled gravitational wave energy spectrum for the R^2-inflation canonical misalignment axion model confronted with the sensitivity curves of gravitational waves experiments for the reheating temperature being low $T_R = 10^4$ GeV.

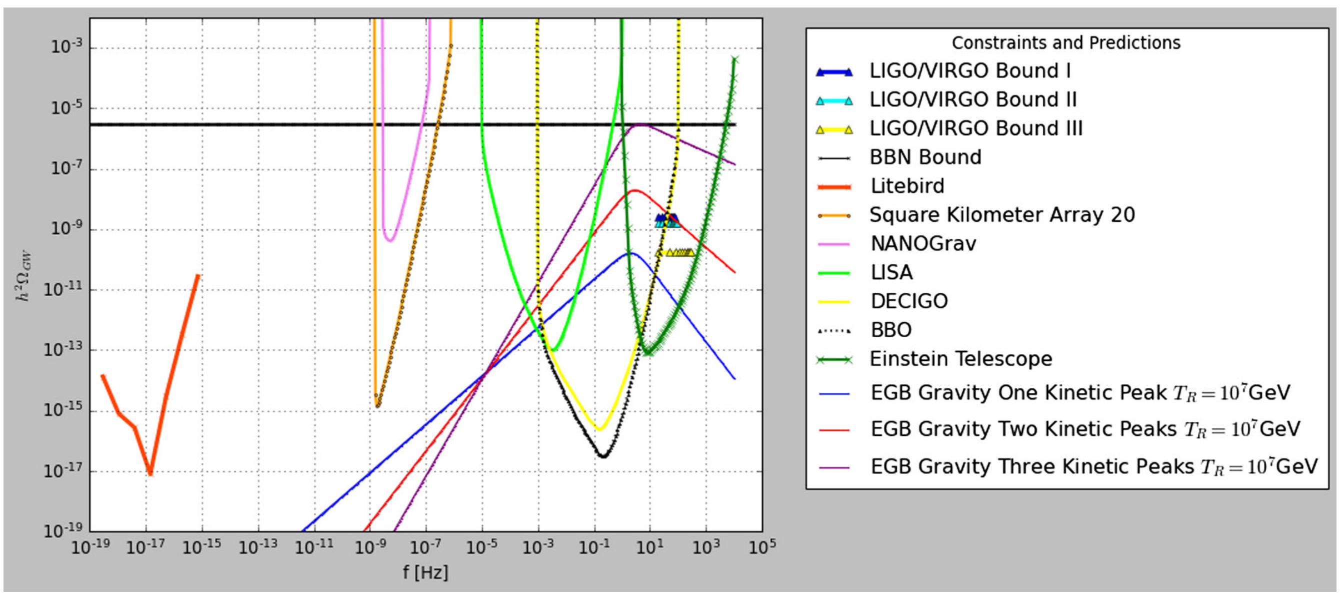

FIG. 7. The h^2-scaled gravitational wave energy spectrum for the Einstein-Gauss-Bonnet model confronted with the sensitivity curves of gravitational waves experiments for the reheating temperature being high $T_R = 10^7$ GeV.

wave energy spectrum are amplified, see for example Fig. 9. However, it is highly unlikely that the background EOS will be modified to such extent during the radiation domination eras by the short kination eras of the axion.

1. Brief comment: Metastable vacuum decay during matter domination era and early dark energy

Let us briefly discuss the implications of the rolling axion towards to its new minimum during the matter domination era. In principle this rolling era can have significant effects on the matter domination era, changing the background EOS parameter from the matter dominating value to something different. The most interesting scenario is the case in which the axion does not roll in a rapid way, but initially slow rolls to its minimum. Since in the matter domination era, the axion plays an important role in the composition of dark matter, an initial slow-roll era might change the background EOS to be de Sitter or even quintessential. Thus it is possible that an early dark energy era is realized in the context of our scenario,

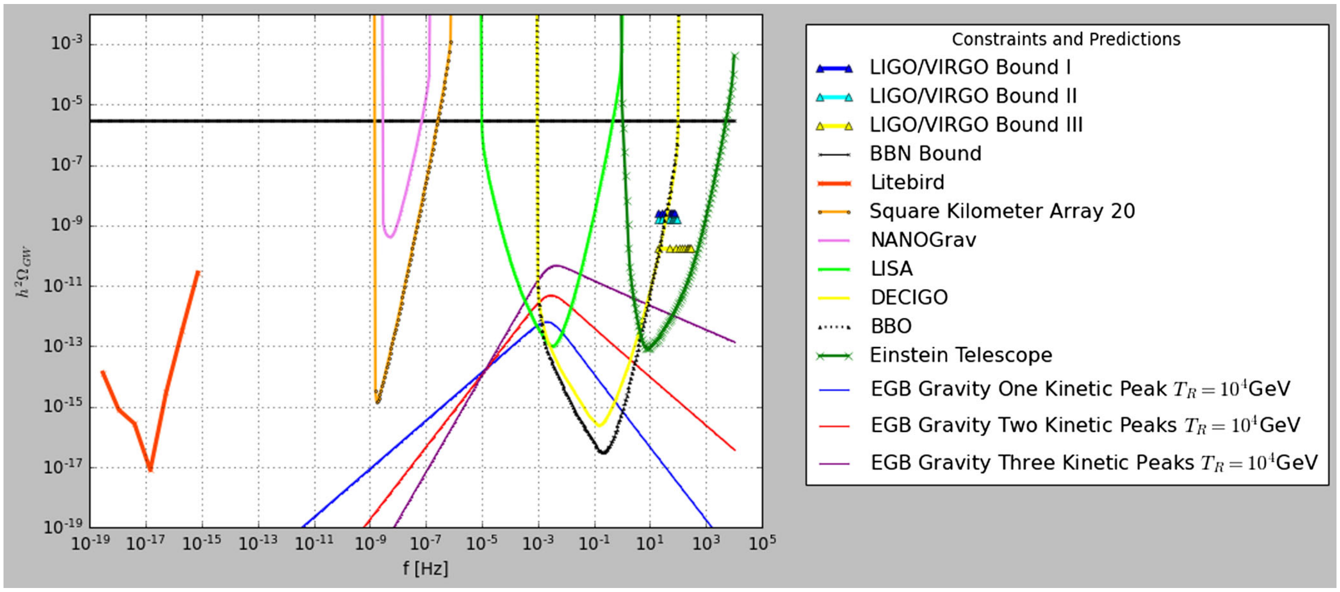

FIG. 8. The h^2-scaled gravitational wave energy spectrum for the Einstein-Gauss-Bonnet model confronted with the sensitivity curves of gravitational waves experiments for the reheating temperature being low $T_R = 10^4$ GeV.

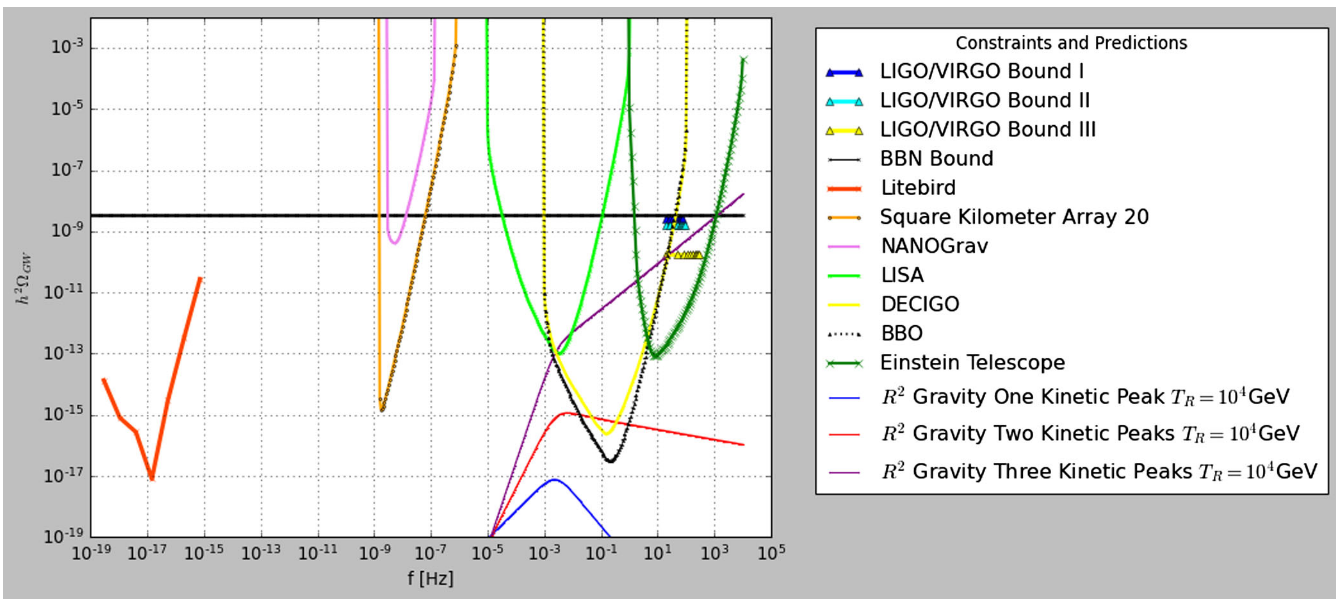

FIG. 9. The h^2-scaled gravitational wave energy spectrum for the R^2 model confronted with the sensitivity curves of gravitational waves experiments for the reheating temperature being low $T_R = 10^4$ GeV, with $w = 0.9$.

and basically not only one, but multiple short dark energy eras are realized, depending on the duration of the unstable axion oscillations at the origin. These early dark energy eras play no role phenomenologically for gravitational wave physics but can explain the H_0 tension, see for example [144]. Of course the axion rolling will change the background EOS parameter from $w = 0$ to $w = 1$ in an abrupt way, so swift kination effects during the matter domination era will take place too. We do not further pursuit this aspect, since it is beyond the purposes of this work.

III. AN EXOTIC SCENARIO: TACHYONIC INSTABILITIES INDUCED IN THE AXION SECTOR VIA THE HIGGS PORTAL

Now we shall consider an exotic scenario according to which the Higgs portal can induce tachyonic instabilities in the axion sector. This is rather unappealing phenomenologically, but we shall consider this for completeness. It should be noted that this mechanism can in principle apply to any other Standard Model singlet axionlike particle coupled to the Higgs and that belong to a hidden sector. In this case, once the axion rendered a tachyon, it can no longer act as dark matter. Thus if the axion is the main component of cold dark matter, then this is a destructive scenario for the axion dark matter perspective Universe. This scenario can only apply in the case that the axion is some part of cold dark matter but not all of it.

The axion-Higgs potential we shall consider in this case is

$$V(H,\phi) = -m_H^2|H|^2 + \lambda_H|H|^4 + \frac{1}{2}m_a^2\phi^2 - \lambda|H|^2\phi^2 + \frac{g}{M^2}\phi^4|H|^2, \tag{35}$$

with $H = \frac{h+ih_1}{\sqrt{2}}$ being the Higgs scalar, also $m_H = 125$ GeV is the Higgs boson mass, λ_H is the Higgs self-coupling, which are related as $\frac{v}{\sqrt{2}} = (\frac{-m_H^2}{\lambda_H})^{\frac{1}{2}}$, where v is the electroweak symmetry breaking scale $v \simeq 246$ GeV. Furthermore, m_a is the axion mass that will be a free parameter in our theory, M is the high scale of the effective theory in which the nonrenormalizable dimension six operator originates from, and λ and g are the coupling of the axion to the Higgs that will be assumed to be extremely small of the order $\lambda \sim \mathcal{O}(10^{-43})$ and $g \sim \mathcal{O}(10^{-35})$, with g being the Wilson coefficient of the higher dimension operator. Thus the renormalizable Higgs portal axion interaction is extremely weak and also the effective axion-Higgs theory at the scale M is also extremely weakly coupled. The weakly coupled nature of the proposed theory generated at high TeV scales is essentially a safe path for Higgs phenomenology, since the branching fraction of the Higgs to the hidden axion sector is severely constrained [35]. Specifically if the branching fraction of the Higgs to invisible axion particles $\mathrm{BR}_{\mathrm{inv}}$ via $h \to \phi\phi$ would be large, then it would be hard to detect the Higgs in the LHC [35]. Also the upper bounds of the branching fraction of the Higgs to the invisible hidden axion sector are $\mathrm{BR}_{\mathrm{inv}} <$ 0.30–0.75 at 95% confidence level (CL) [35]. The potential of Eq. (37) has an inherent Z_2 symmetry $\phi \to -\phi$ that renders the axion particle stable towards decays, since there is no mixing between the Higgs and the axion, and also the axion is considered to be a Standard Model singlet. Such Z_2 singlet-Higgs models are used frequently in the literature and can be related to dark matter components [35,54].

A mentionable feature of the renormalizable Higgs-axion interaction $-\lambda|h|^2\phi^2$ is the negative sign we chose in it. Such negative couplings are particularly favored by Higgs phenomenology. Indeed, the excess of the Higgs decay to diphotons is particularly enhanced for such a negative coupling [35]. Needless to say, the $h \to \gamma\gamma$ channel is very sensitive to new physics, thus the choice for a negative coupling is well motivated. Considering the renormalizable interaction $-\lambda|h|^2\phi^2$, the decay rate of the Higgs to the hidden invisible axion sector is, in our case [54],

$$\Gamma(h \to \phi\phi) = \frac{\lambda^2 v^2}{32\pi m_H}\sqrt{1 - \frac{4m_a^2}{m_H^2}}, \tag{36}$$

so with the axion mass being at the sub-eV range and with the coupling of the renormalizable Higgs-axion interaction being of the order $\lambda \sim \mathcal{O}(10^{-43})$, the decay rate of the Higgs to the hidden axion sector is practically zero. Thus, the branching fraction of the Higgs to Standard Model particles is not affected at all in our case. A large decay rate would affect and practically would reduce the strength of the Higgs boson signal at the LHC, as we already mentioned, but this is not our case.

A confusing question that is raised for the coupling λ entering in the renormalizable Higgs-axion interaction $-\lambda|h|^2\phi^2$ is whether λ is allowed to take such low values, since this would mean that the dark matter particle is out of thermal equilibrium. Indeed, in a standard radiation domination epoch, the thermalization rate is approximately $\Gamma_{\mathrm{th}} \sim \lambda^2 T$ when the temperature is way larger than the Higgs mass; when the temperature drops below the Higgs mass, then $\Gamma_{\mathrm{th}} \sim \lambda^2 T^5 m_H^{-4}$. Thus the ratio of the thermalization rate over the Hubble rate is maximized when $T \sim m_W$, where m_W is the mass of the W boson. The dark matter particle thermalization condition down to the electroweak epoch imposes the constraint $\lambda \geq \sqrt{\frac{m_W}{M_p}} \sim 10^{-8}$ [56]. The caveat in the above considerations however is that the axion is not a thermal relic and is never at thermal equilibrium with the particle content of the Universe, during all the stages of the radiation domination era. Thermal relics are the WIMPs only, which are in thermal equilibrium when the temperature is not larger compared to their mass. When the temperature is larger than the WIMPs masses, these decouple. When the WIMPs are produced, they are produced by particles which are in thermal equilibrium with their background, and the WIMPs energy spectrum is the same with the particles that produces them in thermal equilibrium. On the antipode of this physical picture, axions are not thermal relics and are generated by coherent bosonic motion. The difference between thermal and nonthermal relics are profound, having different relic abundance, masses, and of course couplings. In the case of the axions, there is no restrictive lower bound to the coupling related with thermal equilibrium arguments. In fact, a very weak sector is a desirable feature for the Higgs phenomenology.

Let us proceed to the theory at hand, and the first thing to discuss is the physics of the model at hand. During the electroweak transition the Higgs particle acquires a vacuum expectation value so $H = \frac{v+h+ih_1}{\sqrt{2}}$; therefore the terms that couple the Higgs to the hidden axion sector modify the tree order mass of the axion. Thus, although prior to the electroweak breaking the axion was a normal particle, with an ordinary mass term, after the electroweak breaking the tree order mass is of the form $m_{\mathrm{eff}}^2 = m_a^2 - \lambda v^2$, thus

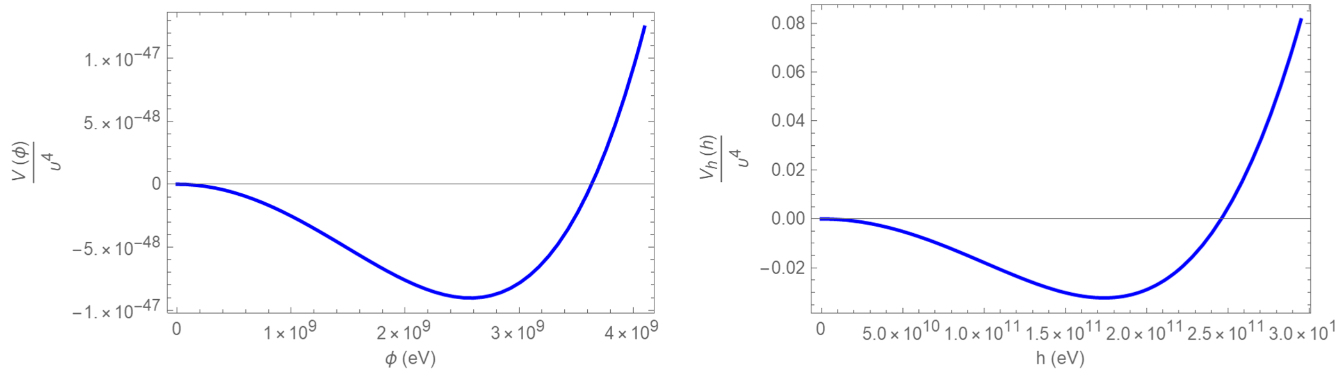

FIG. 10. Tachyonic axion potential (left plot) and the Higgs potential (right plot) after the electroweak symmetry breaking.

depending on the values of λ it is highly possible that the axion turns to a tachyon at the postelectroweak epoch. Indeed, this is the scenario we shall consider. Obviously if $\lambda v^2 \ll ma^2$ the axion never turns into a tachyon and the electroweak epoch will just modify its tree order mass. We shall consider the case in which $\lambda = \frac{2m_a^2}{v^2}$, and since $v =$ 246 GeV and $m_a = 10^{-10}$ eV, it turns out that $\lambda \sim 10^{-43}$. For this value, the tree order potential after the electroweak breaking is well bounded from below, since the inequality $\lambda_H \frac{gv^2}{2M^2} \geq \lambda^2$ is satisfied. For the choice of λ we discussed above, the tree order potential for the axion sector becomes

$$V(\phi) = -\frac{1}{2} m_a^2 \phi^2 + \frac{gv^2}{2M^2} \phi^4. \tag{37}$$

Hence, the axion is basically a tachyon at tree order, and for the values of the parameters chosen as indicated above, the tree order potential can be found in Fig. 10 and the direct comparison with the Higgs potential can be found in the same plot. As it can be seen in Fig. 10, the tachyonic axion potential has a minimum for a field space value different from that at the origin. Thus the tachyonic axion rolls to the new potential minimum. Once it reaches the new minimum, the vacuum state $(h, \phi) = (0, v_s)$ decays to the much more deeper and energetically favorable vacuum state $(h, \phi) = (v, 0)$, in the way described in the previous section. Thus the same physical picture occurs in this case too; however in this case the axion is practically a tachyon, so it is a rather unappealing physical situation. The effect on the gravitational waves though would be the same, with the difference being that a tachyonic instability drives the short kination eras. Finally, the result does not change if one-loop corrections are included in the tachyonic axion tree order potential.

IV. AN IDLE CALCULATION FOR THE SAKE OF COMPLETENESS: INCLUSION OF HIGH TEMPERATURE EFFECTS IN THE AXION SECTOR

As we discussed earlier, the axion particle is not a thermal dark matter relic, such as the WIMPs, but it is a nonthermal relic, generated by coherent bosonic motion. Thus it is senseless to consider high temperature effects on the axion potential at the electroweak phase transition. However, for completeness and just out of curiosity, in this section we shall consider high temperature effects on the axion potential. As is probably expected from the reader, the new minimum present at tree order due to the higher dimensional operators in the axion potential disappear at high temperature, only when two-loop high-temperature or all the higher order daisy graphs are included, and the new minimum reappears once the temperature drops. Thus the physical picture described in the previous sections, in which the axion undergoes several swift kination eras, is delayed if high-temperature effects are considered. But as we already mentioned, this is an idle calculation, the high temperature effects never affect the axion, because the axion is not a thermal relic.

Let us consider in brief the high temperature cases in this section. We start with the physically appealing case in which the new minimum in the axion potential is generated by the dimension six and dimension eight operators. We shall consider one- and two-loop effects in this case. The high temperature corrected effective potential of the axion at one loop, including zero-temperature one-loop effects, has the following form [145]:

$$V_a^{1\text{-loop}T\neq 0} = \mathcal{V}_a(\phi) + V^{1\text{-loop}}(\phi) + \frac{T^4}{2\pi^2} \mathcal{J}_b\left(\frac{m_{\text{eff}}^2(\phi)}{T^2}\right), \tag{38}$$

where the tree order potential $\mathcal{V}_a(\phi)$ and the zero temperature one-loop effective potential $V^{1\text{-loop}}(\phi)$ are defined in Eqs. (4) and (5), respectively. Also the effective mass of the axion $m_{\text{eff}}^2(\phi) = \frac{\partial^2 V(\phi,h)}{\partial \phi^2}$ is defined in Eq. (6). Furthermore, the function $\mathcal{J}_b(\frac{m_{\text{eff}}^2(\phi)}{T^2})$ for small value of $\frac{m_{\text{eff}}^2(\phi)}{T^2}$ [145],

$$\mathcal{J}_b(x) \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12} \frac{m_{\text{eff}}^2(\phi)}{T^2} - \frac{\pi}{6} \frac{m_{\text{eff}}^{3/2}(\phi)}{T^3} - \frac{1}{32} \frac{m_{\text{eff}}^4(\phi)}{T^4} \log\left(\frac{m_{\text{eff}}^2(\phi)}{a_b T^2}\right), \tag{39}$$

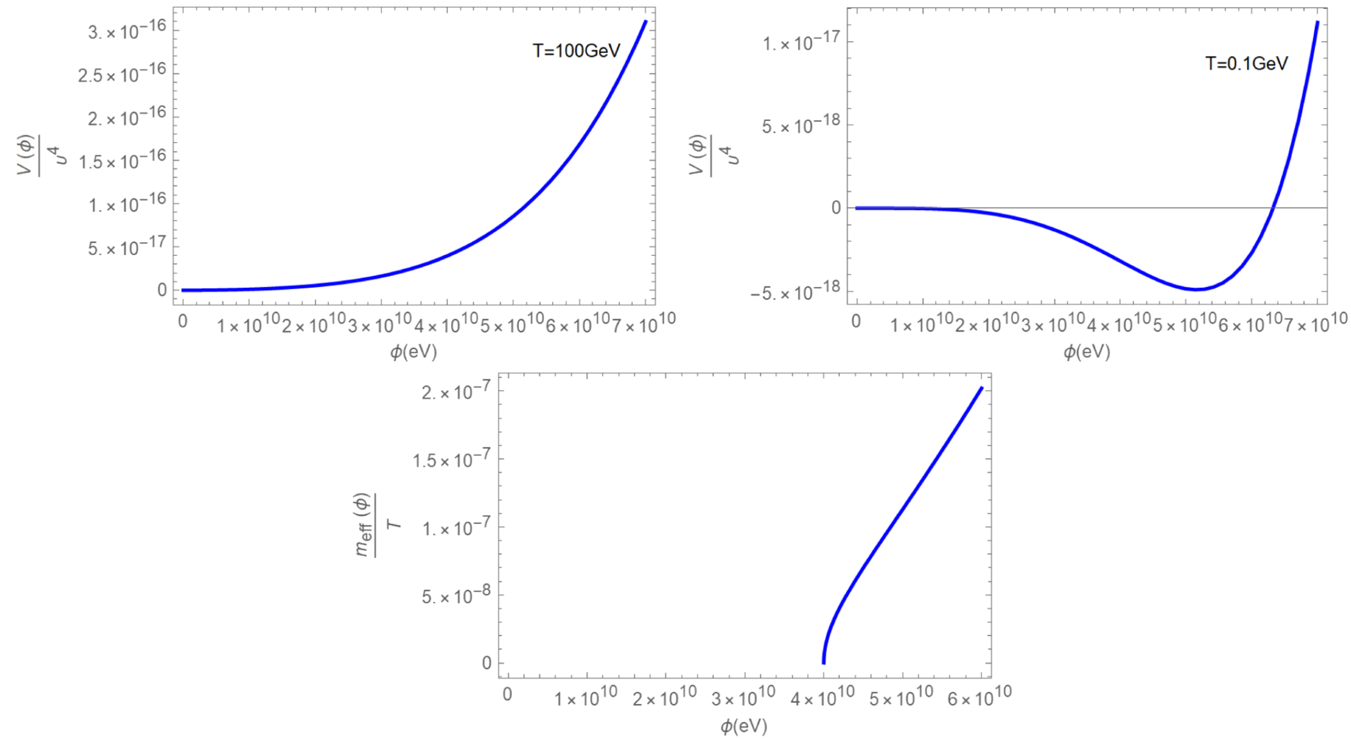

FIG. 11. The high temperature effective potential at two loops for $T = 100$ GeV (upper left plot), the effective potential at $T = 0.1$ GeV (upper right plot), and the behavior of $\frac{m_{\rm eff}(\phi)}{T}$ for all relevant field values (bottom plot) for $T = 100$ GeV which also holds true for $T = 0.1$ GeV. When the temperature is of the electroweak scale order $T \sim 100$ GeV, the axion potential has the true minimum at the origin (upper left plot). The axion potential develops a minimum only when the temperature drops as low as $T \sim 0.1$ GeV (upper right plot).

where $\log a_b = 5.4076$. Thus adding all the contributions, we can see that the effective mass-dependent logarithm terms cancel, and the resulting effective potential in the limit $\frac{m_{\rm eff}^2(\phi)}{T^2} \ll 1$ reads at leading order,

$$\begin{aligned} V_a^{\text{1-loop}T\neq 0} \simeq\, & m_a^2 f_a^2\left(1-\cos\left(\frac{\phi}{f_a}\right)\right) - \lambda\frac{v^2\phi^4}{M^2} + g\frac{v^2\phi^6}{M^6} \\ & + \frac{m_{\rm eff}^4(\phi)}{64\pi^2}\left(\ln\left(\frac{m_{\rm eff}^2(\phi)}{\mu^2}\right)-\frac{3}{2}\right) + \frac{m_{\rm eff}^2(\phi)}{24}T^2 \\ & - \frac{T}{12\pi}(m_{\rm eff}^2(\phi))^{3/2} + \frac{m_{\rm eff}^4(\phi)}{64\pi^2}\left(\log(a_b)-\frac{3}{2}\right) \\ & + \frac{m_{\rm eff}^4(\phi)}{64\pi^2}\log\left(\frac{T^2}{\mu^2}\right), \end{aligned} \tag{40}$$

and, as it proves, the last two terms do not affect the physics corresponding to the above effective potential, when the approximation $\frac{m_{\rm eff}^2(\phi)}{T^2} \ll 1$ holds true. A simple analysis for the above potential, by using the same values for the free parameters as in the tree order potential, indicates that when the temperature is of the order $T \sim 100$ GeV, the axion potential has the same minimum as the tree order potential and this said behavior continues as the temperature drops. For all the temperatures and field values, the approximation $\frac{m_{\rm eff}^2(\phi)}{T^2} \ll 1$ always holds true. This is peculiar, so we included the two-loop correction to the axion effective potential at high temperature $\sim \frac{gv^2}{2M^4}\phi^2 T^4$ [146], and for $\lambda \sim \mathcal{O}(10^{-10})$ an interesting behavior occurs. Specifically, the axion minimum at $T \sim 100$ GeV is at the origin, and when the temperature drops as low as $T \sim 0.1$ GeV, the axion potential develops the second minimum, which is not (yet) identical to the zero-temperature one. The new minimum becomes identical to the tree order minimum when the temperature effects are no longer dominant. We need to note that there is no barrier between the local maximum at the origin and the new local minimum. Thus from the perspective of phase transitions, this behavior mimics a second order phase transition. For all the temperatures and field values, the approximation $\frac{m_{\rm eff}^2(\phi)}{T^2} \ll 1$ always holds true. This said behavior is depicted in the three plots of Fig. 11, where we plot the high temperature effective potential at one-loop for $T = 100$ GeV (upper left plot), the effective potential at $T = 0.1$ GeV (upper right plot), and the behavior of $\frac{m_{\rm eff}(\phi)}{T}$ for $T = 100$ GeV for all relevant field values (bottom plot) that also hold true for $T = 0.1$ GeV. Regarding the tachyon axion case, we shall include all the higher order loop graphs, the so-called daisy contributions at all orders, and the tachyon axion effective potential in this case reads,

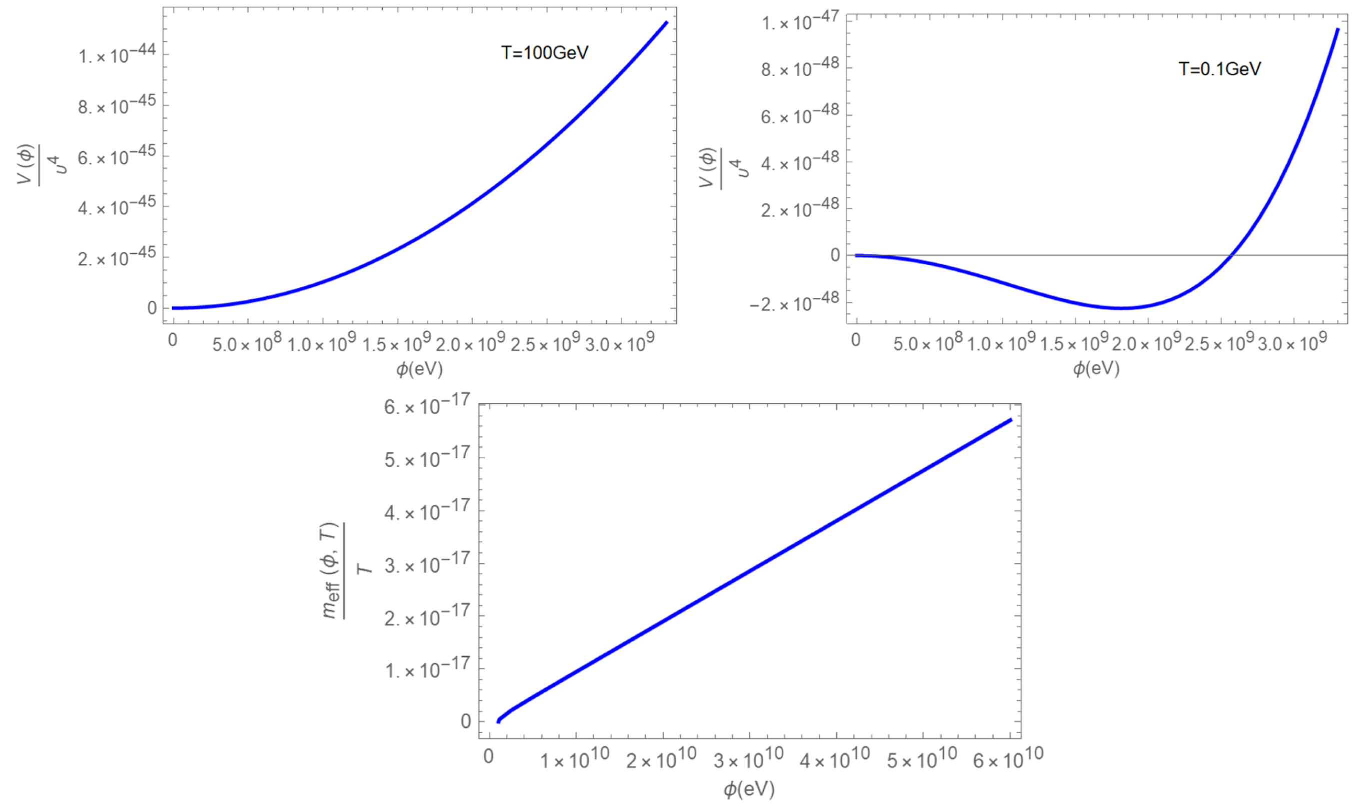

FIG. 12. The high temperature effective potential for all daisy graphs, for the tachyonic axion case, for $T = 100$ GeV (upper left plot), the daisy resumed effective potential at $T = 0.1$ GeV (upper right plot) and the behavior of $\frac{m_{\rm eff}(\phi,T)}{T}$, for all relevant field values (bottom plot) for $T = 100$ GeV which also holds true for $T = 0.1$ GeV. When the temperature is of the electroweak scale order $T \sim 100$ GeV, the tachyon axion potential has the true minimum at the origin (upper left plot). The axion potential develops a minimum only when the temperature drops as low as $T \sim 0.1$ GeV (upper right plot).

$$V_a^{{\rm daisy}T\neq 0} \simeq -\frac{1}{2}m_a^2\phi^2 + \frac{gv^2}{2M^2}\phi^4 + \frac{gv^2}{2M^2}T^2 - \frac{1}{12\pi}\left(-m_a^2 + \frac{gv^2}{2M^2}T^2 + \frac{6gv^2}{M^2}\phi^2\right)^{3/2}T. \tag{41}$$

In this case too, only when $g \sim \mathcal{O}(10^{-35})$, when the temperature is as high as $T \sim 100$ GeV, then the minimum of the tachyonic axion potential disappears and reappears at $T \sim 0.1$ GeV, as it can be seen in the left and right upper plots of Fig. 12. In the bottom plot we present the behavior of $m_{\rm eff}(\phi,T)/T$, where $m_{\rm eff}(\phi,T) = \frac{T^2(gv^2)}{2M^2} + \frac{12\phi^2(gv^2)}{2M^2} - m_a^2$. As it can be seen the high temperature approximation holds true for all the relevant field values, as long as the perturbation theory approximation is not violated.

Thus, in the case the high temperature corrections are included, the qualitative picture of the tree order case does not change, it is only delayed.

V. CONCLUSIONS AND DISCUSSION

In this work we considered the effects of short axion kination eras on the energy spectrum of the primordial gravitational waves for modes reentering the Hubble horizon near the electroweak symmetry breaking epoch and beyond, well inside the radiation domination era. We assumed that the axion particle is coupled to the Higgs particle solely via higher dimensional operators, of dimension six and eight. When the electroweak symmetry breaking epoch occurred, these higher dimensional operators modified the axion potential, causing a new minimum to it and thus destabilizing the minimum at the origin. Once the axion oscillations at the origin are destabilized after some time, the axion starts to swiftly roll its potential heading to the new minimum, experiencing a kinetic evolution, with its energy density redshifting as $\rho_a \sim a^{-6}$. Depending on whether the axion composes the whole or some part of the dark matter, this kinetic evolution may affect to some extent the total background EOS, changing it from the radiation value $w = 1/3$ to some value closer to the kination domination value $w = 1$. We chose a conservative approach and assumed that the background EOS parameter value changed to $w = 0.5$. When the axion reached its new potential minimum, due to the fact that the Higgs electroweak vacuum is energetically more favorable than the axion minimum, the latter decays to the Higgs vacuum and the Universe is again described by the latter. Thus the axion returns to the origin of its potential, and due to the quantum fluctuations it starts its oscillations, due to the dominance of the potential term $\sim\phi^2$. After some considerable time the oscillations are disrupted and the axion starts to roll towards in its new minimum and the procedure we described is repeated again. Thus during the radiation domination era these short axion kination epochs may occur many times. These deformations of the background EOS parameter may have measurable effects on the primordial gravitational waves. These effects mainly depend on how many times the short in duration axion kination epochs occur, on the theory that describes the inflationary era, the background EOS parameter value during the short kination era and the reheating temperature. We investigated several scenarios, disregarding the primordial gravitational waves coming from the electroweak phase transition, and the results we found indicate a characteristic structure in the energy spectrum that can be observed in future gravitational wave experiments. Finally, we considered an alternative scenario with a direct renormalizable coupling between the Higgs and the axion. In this scenario, the axion is rendered a tachyon during and beyond the electroweak breaking epoch, but the same physical behavior occurs as in the case of the dimensions six and dimension eight nonrenormalizable operators. Finally, we included a purely academic study, including high temperature effects in the axion potential. This is not physically justified though, since the axion never thermalizes with its background, since it is a nonthermal relic that is generated by coherent bosonic oscillations. However, just out of curiosity we included this study, and, as we demonstrated, the temperature effects delay the commencing of the short kination eras, but the same physics occur nonetheless. We also discussed the effects of this axion rolling scenario on the matter domination epoch. During the latter, the effects would be more dramatic, and we examined the case that the axion does not roll swiftly from the beginning of its motion, but it slow rolls initially. This would cause multiple short early dark energy epochs during the matter domination epoch. A future perspective could be to include several other singlet axionlike particles in this scenario and investigate the overall effect on the short kination eras due to the electroweak symmetry breaking. Apparently, the whole scenario is based on the fact that the reheating temperature was higher than $T \sim 100$ GeV, and that the electroweak epoch occurred during the radiation domination era. However, this is not certain, since the reheating era may have not been so large [112]. Thus one should consider alternative scenarios that may control the electroweak symmetry breaking epoch. We hope we shall address these aspects in some future works. Finally, before closing there are a lot to say about the shape of the predicted gravitational wave energy spectrums of different models regarding the slope and the peak frequency. There are a lot to say at this point, on how to discriminate these curves and pinpoint the correct model, and also one must also take into account the direct effect of the reheating temperature, the frequency range that the peak is observed, how wide the peak is, the peak and curve observed by one or multiple detectors, and how would the effects of phenomena we ignored, like primordial gravitational waves or the effects of a first order phase transition, would modify the above picture. Undoubtedly, these will be very important issues that theoreticians should discuss and collaborate on before the LISA and Einstein telescope yield their first observational data on primordial stochastic gravitational waves.

ACKNOWLEDGMENTS

This research has been is funded by the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP14869238).

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