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Exact solutions for the (3+1)-dimensional Kudryashov-Sinelshchikov equation

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Abstract. In this work, (3+1)-dimensional Kudryashov-Sinelshchikov equation is investigated by using the sine-cosine method and modification of the truncated expansion method. A variety of exact solutions are obtained.

1. Introduction
In this paper, we study the (3+1)-dimensional Kudryashov-Sinelshchikov equation [1]

$$(u_t + \alpha u u_x + \gamma u_{xxx})_x + d u_{yy} + e u_{zz} = 0, \tag{1}$$

where α represents the nonlinearity, γ is dispersion term, while d and e stand for transverse variation of wave in y and z directions. The equation (1) describes the physical characteristics of nonlinear waves in a bubbly liquid. In the case $e = 0$, equation (1) reduces to two-dimensional Korteweg-de Vries equation and for case $d = e = 0$ we obtain the one-dimensional Korteweg-de Vries equation. The equation (1) was studied by the modified tanh-coth method [2], Backlund transformation [3], bifurcation analysis was presented in [4], and density fluctuation symbolic computation in [5]. In one-dimensional and two-dimensional cases the Kudryashov-Sinelshchikov equations were studied by the G'/G expansion method in [6], the first integral method was applied in [7], modification of truncated expansion method [8], the modified exp-function method [9].

The purpose of this work is to find exact solutions for the (3+1)-dimensional Kudryashov-Sinelshchikov equation. The methods for finding exact solution of nonlinear partial differential equations are known. Some of them are the Darboux transformation [10–13], Hirota bilinear method [14–17], Kudryashov method [18], extended tanh method [19, 20], sine-cosine method [20]. To obtain the exact solution for the (3+1)-dimensional Kudryashov-Sinelshchikov equation we use the two methods such as the sine-cosine method [20] and the modification of the truncated method [8].

The organization of the paper is as follows: In Section 2, the description of the sine-cosine method and exact solution are given. In section 3, we study the Kudryashov-Sinelshchikov equation by the modification of the truncated method. Finally, the conclusion is given in Section 4.

2. The Sine-cosine method

2.1. Review of the sine-cosine method

In this section we describe sine-cosine method that is presented in [20]. According to method the partial differential equation

$$E_1(u_t, u_x, u_{xx}, u_{xxx}, ...) = 0, \tag{2}$$

can be converted to ODE

$$E_2(u, u', u'', u''', ...) = 0, \tag{3}$$

by using a wave variable

$$u(x,t) = u(\xi), \quad \xi = x - ct. \tag{4}$$

Then equation (3) is integrated as long as all terms contain derivatives where integration constants are considered zeros. The solutions of ODE can be expressed in the form

$$u(x,t) = \lambda \cos^{\beta}(\mu\xi), \quad |\xi| \le \frac{\pi}{2\mu}, \tag{5}$$

or

$$u(x,t) = \lambda \sin^{\beta}(\mu\xi), \quad |\xi| \le \frac{\pi}{\mu}, \tag{6}$$

where the parameters λ, μ and β will be determined, and μ is wave number and c is wave speed respectively. The derivatives of (5) become

$$(u^n)_{\xi} = -n\beta\mu\lambda^n \cos^{n\beta-1}(\mu\xi)\sin(\mu\xi), \tag{7}$$

$$(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \tag{8}$$

and the derivatives of (6) have next forms

$$(u^n)_{\xi} = -n\beta\mu\lambda^n \sin^{n\beta-1}(\mu\xi)\cos(\mu\xi), \tag{9}$$

$$(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi), \tag{10}$$

and so on for the other derivatives. Using (5)-(10) into the reduced ODE gives a trigonometric equation of $\cos^R(\mu\xi)$ or $\sin^R(\mu\xi)$ terms. Then, we determine the parameters by first balancing the exponents of each pair of cosine or sine to determine R. Next, we collect all coefficients of the same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$, where these coefficients have to vanish. The system of algebraic equations among the unknown β, λ, and μ will be given and from that, we can determine coefficients.

2.2. Application the sine-cosine method

In this section, we apply sine-cosine method to the (3+1)-dimensional equation (1). By wave variable

$$u(x,y,z,t) = u(\xi), \quad \xi = (x + y + z - ct), \tag{11}$$

the equation (1) can be converted to

$$(-c + d + e)u + \frac{\alpha}{2}u^2 + \gamma u'' = 0. \tag{12}$$

Seeking the solution in (5) and (8)

$$\begin{aligned}&\cos^{\beta}(\mu\xi)\left[(-c+d+e)\lambda-\gamma\mu^2\beta^2\lambda\right]+\\&+\frac{\alpha}{2}\lambda^2\cos^{2\beta}(\mu\xi)+\mu^2\lambda\beta(\beta-1)\cos^{\beta-2}(\mu\xi)=0.\end{aligned}\tag{13}$$

Equating the exponents and the coefficients of each pair of the $\cos(\mu\xi)$ functions we find the following algebraic system:

$$2\beta=\beta-2,\rightarrow\beta=-2.\tag{14}$$

Substituting equation (14) into equation (13) to get

$$\begin{aligned}&\cos^{-2}(\mu\xi)\left[(-c+d+e)\lambda-\gamma\mu^2\beta^2\lambda\right]+\\&+\frac{\alpha}{2}\lambda^2\cos^{-4}(\mu\xi)+\mu^2\lambda\beta(\beta-1)\cos^{-4}(\mu\xi)=0.\end{aligned}\tag{15}$$

Equating the exponents and the coefficients of each pair of the $\cos(\mu\xi)$ functions, we obtain a system of algebraic equations

$$\cos^{-2}(\mu\xi)\ :\ (-c+d+e)\lambda-\gamma\mu^2\beta^2\lambda=0,\tag{16}$$

$$\cos^{-2}(\mu\xi)\ :\ \frac{\alpha}{2}\lambda^2+\mu^2\lambda\beta(\beta-1)=0.\tag{17}$$

Solving the algebraic system (16)-(17), we get:

$$\lambda=\frac{-12\mu^2}{\alpha},\quad \mu=-\frac{1}{2}\sqrt{\frac{d+e-c}{\gamma}}.\tag{18}$$

The result (18) can be easily obtained if we also use the sine method (6) and then we obtain the following exact solutions

$$u_1(x,y,z,t)=\frac{-12\mu^2}{\alpha}\cos^{-2}\left(\frac{1}{2}\sqrt{\frac{d+e-c}{\gamma}}(x+y+z-ct)\right),\tag{19}$$

$$u_2(x,y,z,t)=\frac{-12\mu^2}{\alpha}\sin^{-2}\left(\frac{1}{2}\sqrt{\frac{d+e-c}{\gamma}}(x+y+z-ct)\right),\tag{20}$$

where $c\neq d+e$.

3. Modification of the truncated expansion method

3.1. Review modification of the truncated expansion method

The partial differential equation

$$E_1(u_t,u_x,u_{xx},u_{xxx},...)=0,\tag{21}$$

can be converted to ODE

$$E_2(u,u',u'',u''',...)=0,\tag{22}$$

by using a wave variable

$$u(x,t)=u(\xi),\quad \xi=x-ct.\tag{23}$$

To find dominant terms we substitute

$$u = \xi^{-p}, \tag{24}$$

into all terms of equation (22). Then we ought to compare degrees of all terms of equations and choose two or more with the highest degree. The maximum value of p is called the pole of the equation (22) and we denote it as N. The method can be applied when N is integer. The exact solution of equation (22) is looked in the form

$$u = a_0 + a_1 Q(\xi) + a_2 Q(\xi)^2 + ... + a_N Q(\xi)^N, \tag{25}$$

where $Q(\xi)$ is the following function

$$Q(\xi) = \frac{1}{1+e^{\xi}}. \tag{26}$$

We can calculate number of derivatives by

$$u_\xi = \sum_{n=0}^{N} a_n n Q^n (Q-1), \tag{27}$$

$$u_{\xi\xi} = \sum_{n=0}^{N} a_n n Q^n (Q-1)[(n+1)Q - n], \tag{28}$$

$$u_{\xi\xi\xi} = \sum_{n=0}^{N} a_n n Q^n (Q-1)[(n^2+3n+2)Q^2 - (2n^2+3n+1)Q + n^2]. \tag{29}$$

3.2. Application to Kudryashov-Sinelshchikov Equation

In this section, we apply modification of truncated expansion method to the (3+1)-dimensional Kudryashov-Sinelshchikov equation (1). By wave variable

$$u(x,y,z,t) = u(\xi), \quad \xi = (x+y+z-ct), \tag{30}$$

the equation (1) can be converted to ODE

$$(-c+d+e)u + \frac{\alpha}{2}u^2 + \gamma u'' = 0. \tag{31}$$

From equation (31) we find $N = 2$ then we look for the solution of equation (31) in the form

$$u = a_0 + a_1 Q(\xi) + a_2 Q(\xi)^2. \tag{32}$$

The second derivative of equation (32) is

$$u_{\xi\xi} = a_1 Q + (4a_2 - 3a_1)Q^2 + (2a_1 - 10a_2)Q^3 + 6a_2 Q^4. \tag{33}$$

Substituting (32)-(33) into (31) we obtain the system of algebraic equations in the form

$$Q^4 : \frac{1}{2}a_2^2\alpha + 6a_2\gamma, \tag{34}$$

$$Q^3 : a_1 a_2 \alpha + 2a_1\gamma - 10a_2\gamma, \tag{35}$$

$$Q^2 : -a_2 c + a_2 d + a_2 e + a_0 a_2 \alpha + \frac{1}{2}a_1^2\alpha - 3a_1\gamma + 4a_2\gamma, \tag{36}$$

$$Q^1 : a_0 a_1 \alpha - a_1 c + a_1 d + a_1 e + a_1\gamma, \tag{37}$$

$$Q^0 : -a_0 c + a_0 d + a_0 e + \frac{1}{2}a_0^2\alpha. \tag{38}$$

From the system (34)-(38) we can find coefficients with two cases as

$$1)\ a_0 = 0, \quad a_1 = \frac{12\gamma}{\alpha}, \quad a_2 = -\frac{12\gamma}{\alpha}, \quad c = d + e + \gamma, \tag{39}$$

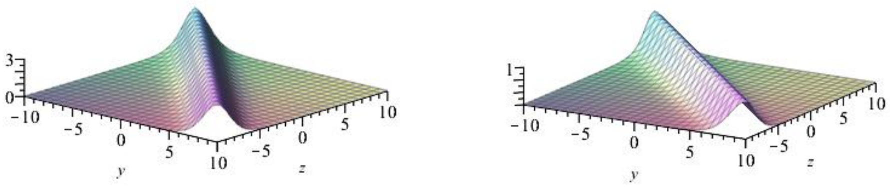
$$2)\ a_0 = -\frac{2\gamma}{\alpha}, \quad a_1 = \frac{12\gamma}{\alpha}, \quad a_2 = -\frac{12\gamma}{\alpha}, \quad c = -\gamma + d + e. \tag{40}$$

Substituting (39)-(40) in (32) we obtain exact solutions of equations (1) in the form

$$u_3(x,y,z,t) = \frac{12\gamma}{\alpha}\frac{1}{1+e^{\xi}} - \frac{12\gamma}{\alpha}\left(\frac{1}{1+e^{\xi}}\right)^2, \tag{41}$$

$$u_4(x,y,z,t) = -\frac{2\gamma}{\alpha} + \frac{12\gamma}{\alpha}\frac{1}{1+e^{\xi}} - \frac{12\gamma}{\alpha}\left(\frac{1}{1+e^{\xi}}\right)^2, \tag{42}$$

where $\xi = (x + y + z - ct)$. The graphical representation of u_3 and u_4 is shown in Fig. 1



a) b)

Figure 1: Solutions corresponding to u_3 and u_4 for $\alpha = 1, \gamma = 1, d = 0.5, e = 0.5, x = 0, t = 0$.

4. Conclusion

In this paper, the (3+1)-dimensional Kudryashov-Sinelshchikov equation was studied using the two methods such as the sine-cosine method and the modification of the truncated method. The schemes of the two methods were presented. New exact solutions for the Kudryashov-Sinelshchikov equation were obtained. These methods can be applied to other kinds of nonlinear problems.

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