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Fourier Transformation method for solving integral equation in the 2.5D problem of electric sounding

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Fourier Transformation method for solving integral equation in the 2.5D problem of electric sounding

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Abstract. The paper discusses a method for solving an integral equation for calculating a three-dimensional electric field in a medium with a two-dimensional geometry based on the Fourier transform. The results of the numerical solution of the transformed integral equation and the original integral equation for the medium with the surface relief are compared. The original equation was solved using parallelization technologies on a system with shared memory. A significant performance improvement based on the transformed equations, including in comparison with the parallel version of the program for the original integral equation, is shown.

1. Introduction

One of the historically the first methods in the study of the earth's interior is the direct current sounding method [1] - [2]. This method remains relevant to this day, since it provides greater depth and mathematical methods for interpreting the results are quite developed [3]-[12] . The development of this method is realized by improving the equipment, automating the measurement process and the ability to carry out measurements with high density [13]-[14]. The mathematical model is based on Maxwell's equations for direct current in a continuous medium. The closing relation for the model is Ohm's law, according to which the current density in the medium is the product of the specific conductivity of the medium and the gradient of the potential of the electric field. For the first time, the study of the effect of the relief on the results of interpretation was carried out in [9] - [12] by the methods of finite elements and finite differences. This study continues the themes of [15] - [17]. The novelty in this work is the use of the direct and inverse cosine Fourier transforms for the numerical solution of an integral equation arising in the problem of sounding a medium with a surface relief.

2. The mathematical model

The mathematical algorithm for solving the quasi-three-dimensional problem described in this paper is the Fourier transform method that was developed in 1987 [18]. The idea of the method is also described in more detail in the monograph [17]. Based on this algorithm, programs such as IE2DR1 and PRIZT, widely used in geophysical practice. However, these programs do not provide for the possibility of modeling environments with a surface relief. This is due to the fact that the basis of the solved integral equations [17] is the reflection method, which works for a flat surface of the medium. Based on the idea of this method, in this paper, we simulated the electric field for the problem electrical tomography over conductive medium with a relief surface.

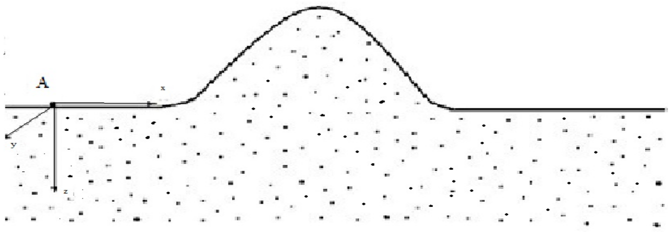


Figure 1. The model of the relief of the day surface, A - the position of the suply electrode

Given a consistent relief (Fig. 1). In this case we write the integral equation for the density of the secondary charges q_0 (M), modeled using so-called simple layer [17]:

$$q_0\left(M\right)=\frac{1}{2\pi}\frac{\partial}{\partial n^0}\left(\frac{1}{|AM|}\right)-\frac{1}{2\pi}\iint_{\Gamma^0}q_0\left(P\right)\frac{\partial}{\partial n_M^0}\left(\frac{1}{|PM|}\right)d\Gamma_p^0 \tag{1}$$

The task is two-dimensional in geometry of the medium, but three-dimensional in terms of the source. Therefore, the desired electric field is three-dimensional. However, the integral equation (1) allows reducing the problem to a two-dimensional one. Further, the solution of the problem in such a medium can be obtained by directly solving equation (1), as was done, for example, in [15] - [16]. However, the Fourier transform method makes it possible to further reduce the dimension of equation (1) to unity. This makes it possible to reduce by an order of magnitude the computation time of the electric field of a point source in two-dimensional media.

Apply this method to the integral equation for the homogeneous relief (1).

The suply electrode is located at $A(x_A, y_A, z_A)$ - this is an electrode that excites a field of our research with a direct current. In the method of integral equations, the electric field is modeled by the sum of the primary field of the feeding electrode in a homogeneous half-space and the field of secondary sources located on the surface of the relief.

Let's pretend that $M(x, y, z)$, and the coordinates of the point $P(x', y', z')$. Green's function $G(P, M) = \left(\frac{1}{|PM|}\right)$ depends on the following arguments (x, x', y, y', z, z'). In equation (1), this function is differentiated by the direction of the normal at the point $M(x, y, z)$. For a two-dimensional relief, elongated along the y axis, the direction of the normal depends only on the x and z coordinates of the point (x, y, z). This means this function depends on the coordinates $x, x', y - y', z, z'$.

It can be noted that integration over the surface Γ^0 can be represented as sequential integration along a forming line directed along axis, and then along the contour of L.

Thus, formula (1) can be written in the form:

$$q_0\left(x,y,z\right)=\frac{1}{2\pi}\frac{\partial}{\partial n_M^0}G\left(x,x_A,y-y_A,z,z_A\right)$$
$$-\frac{1}{2\pi}\int_L\int_{-\infty}^{+\infty}q_0\left(x',y',z'\right)\frac{\partial}{\partial n_M^0}G\left(x,x',y-y',z,z'\right)dy' dL \tag{2}$$

The internal integral in equation (2) is the convolution integral of functions $q_0\left(x', y', z'\right)$ and $\frac{\partial}{\partial n_M^0}G\left(x, x', y - y', z, z'\right)$ on the coordinate of [17] .

Now we go into the spectral region. Since the functions $q_0\left(x, y, z\right)$, $q_0\left(x', y', z'\right)$, $\frac{\partial}{\partial n_M^0}G\left(x, x', y - y', z, z'\right)$ and $\frac{\partial}{\partial n_M^0}G\left(x, x_A, y - y_A, z, z_A\right)$ are even functions with relative to the variable y, apply the direct Fourier cosine transform [18]-[21] . So:

$$\tilde{q}_0\left(x',k_y,z'\right) = 2\int_0^\infty q_0(x',y',z')\cos\left(k_y\cdot y'\right)dy$$

$$\frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x',k_y,z,z'\right) = 2\int_0^\infty \frac{\partial}{\partial n_M^0}G\left(x,x',y-y',z,z'\right)\cos\left(k_y\cdot y\right)dy$$

$$\frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x_A,k_y,z,z_A\right) = 2\int_0^\infty \frac{\partial}{\partial n_M^0}G\left(x,x_A,y-y_A,z,z_A\right)\cos\left(k_y\cdot y\right)dy$$

Since the medium is two-dimensional, the normal n does not depend on the y coordinate, therefore the Fourier transform and differentiation are permutable. Spectra $\tilde{q}_0$, $\frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x_A,k_y,z,z_A\right)$ $and \frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x',k_y,z,z'\right)$ are amplitudes of spatial harmonics with frequency ky. Then the Fourier cosine transform of the integral equation (2) is written in the form:

$$\tilde{q}_0\left(x,k_y,z\right) = \frac{1}{\pi}\int_0^\infty \frac{\partial}{\partial n_M^0}G\left(x,x_A,y-y_A,z,z_A\right)\cos\left(k_y\cdot y\right)dy - \\ -\frac{2}{\pi}\int_L\int_0^\infty \left[q_0(x',y',z')\frac{\partial}{\partial n_M^0}G\left(x,x',y-y',z,z'\right)\right]*\cos\left(k_y\cdot y\right)dy\,dL \quad (3)$$

The convolution spectrum is the product of the spectra of convoluted functions. Consequently,

$$\tilde{q}_0\left(x,k_y,z\right) = \frac{1}{\pi}\frac{\widetilde{\partial}}{\partial n^0}G\left(x,x_A,k_y,z,z_A\right) - \frac{2}{\pi}\int_L \tilde{q}_0\left(x',k_y,z'\right)\frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x',k_y,z,z'\right)dL \quad (4)$$

Integral equation (4) for the spectra of secondary sources on the frequency ky written not on the surface Γ^0, as in the original equation, and along the contour L. Thus, equation (4) is the Fourier cosine transform of integral equation (2), which has a smaller dimension in the spatial variable.

For a homogeneous half-space we have:

$$\frac{\partial}{\partial n_M^0}G\left(x,x',y-y',z,z'\right) = \frac{\rho}{\pi r^3}\mathbf{r}n$$

$$\frac{\partial}{\partial n_M^0}G\left(x,x_A,y-y_A,z,z_A\right) = \frac{\rho}{\pi r^3}\mathbf{r}n$$

where n is the unit vector of the outer normal to the surface Γ^0 at the point (x, y, z). Taking into account the fact that (1y · n) = 0, for the spectra $\frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x',k_y,z,z'\right)$ and $\frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x_A,k_y,z,z_A\right)$ we get the following expressions:

$$\frac{\widetilde{\partial}}{\partial n_M^0}G\left(x,x',k_y,z,z'\right) = \frac{1}{\pi}\left[\left(x-x'\right)\cdot(n_x) + \left(z-z'\right)(n_z)\right]\int_0^\infty \frac{\cos\left(k_y\cdot y\right)}{\left(R^2+y^2\right)^{3/2}}dy \quad (5)$$

$$\frac{\widetilde{\partial}}{\partial n^0}G\left(x,x_A,k_y,z,z_A\right) = \frac{1}{\pi}\left[\left(x-x_A\right)(n_x) + (z)(n_z)\right]\int_0^\infty \frac{\cos\left(k_y\cdot y\right)}{\left(R_A^2+y^2\right)^{3/2}}dy \quad (6)$$

Where $R^2 = \left(x - x^{'}\right)^2 + \left(z - z^{'}\right)^2$, $R_A^2 = (x - x_A)^2 + (z - z_A)^2$. Values R, R_A-projections on the xo axis respectively r, r_A.

In formulas (5) - (6) there is a cosine transform of the functions of the form $\frac{I}{(a^2+y^2)^{3/2}}$. For this transformation we have:

$$S\left(k_y, a\right) = \int_0^\infty \frac{\cos\left(k_y \cdot y\right)}{\left(a^2+y^2\right)^{3/2}} dy = \frac{k_y}{2a} K_1(a \cdot k_y) \tag{7}$$

Where $K_1(x)$ -Macdonald function (modified Bessel function of the 2nd kind) of the first order. In the numerical solution of the integral equation (6) to calculate $\frac{\partial}{\partial n_M^0}\left(\frac{1}{|PM|}\right)$ and $\frac{\partial}{\partial n_M^0}\left(\frac{1}{|AM|}\right)$in a homogeneous half-space, tables of function values were used $K_1(x)$. In order for equation (4) to be reduced to a SLAE, the contours L are divided into elements $\triangle l$ which $q_0\left(x, y, z\right)$ are considered permanent. After the spectral density of the secondary sources is found l_0, go to spatial variables using the inverse cosine Fourier transform:

$$q_0(x, y, z) = \frac{1}{\pi} \int_0^\infty \widetilde{q_0}(x, k_y, z) \cos(k_y \cdot y) dk_y \tag{8}$$

Further, using the found density of secondary charges, we calculate the potential of the electric field.

3. Numerical results

The numerical solution of the integral equation was carried out by discretization of formulas (4) and (8) on a logarithmic grid in frequency. To calculate the cosine - Fourier transform, we restrict ourselves to the final part of the boundary Γ^0. A uniform grid was built on this part of the border. The calculation of the field potential was carried out at points corresponding to the location of the measuring electrodes. Then, through the potential difference of the field, the apparent resistivity of the medium was calculated, as is customary in geophysical experiments.

According to the available geophysical data, if the supply electrode is sufficiently removed from the relief structure, the curves of apparent resistivity should have a symmetrical character. We used this data to test the program, changing the angle of slope of the relief and obtained the corresponding symmetric apparent resistivity. Calculations were carried out for the slope angles of the relief at values of 5, 10, 15 degrees and the curves of apparent resistivity corresponding to them were obtained (Fig. 2 , Fig. 3).

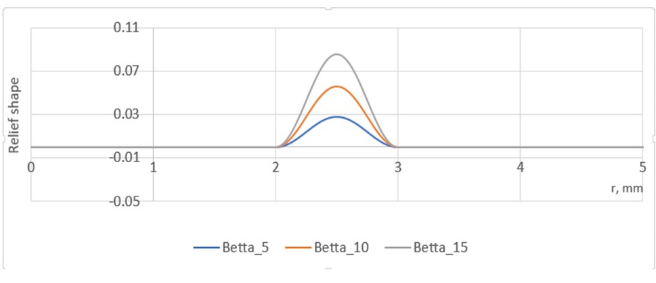


Figure 2. Tilt angle of a relief at values 5, 10, 15 degrees

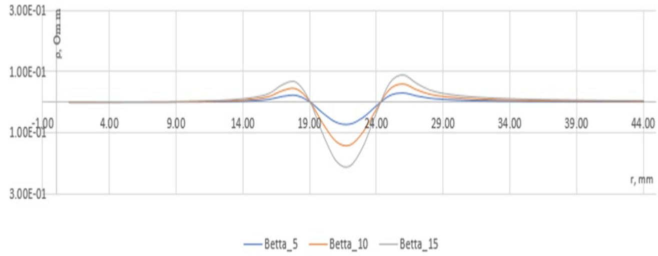
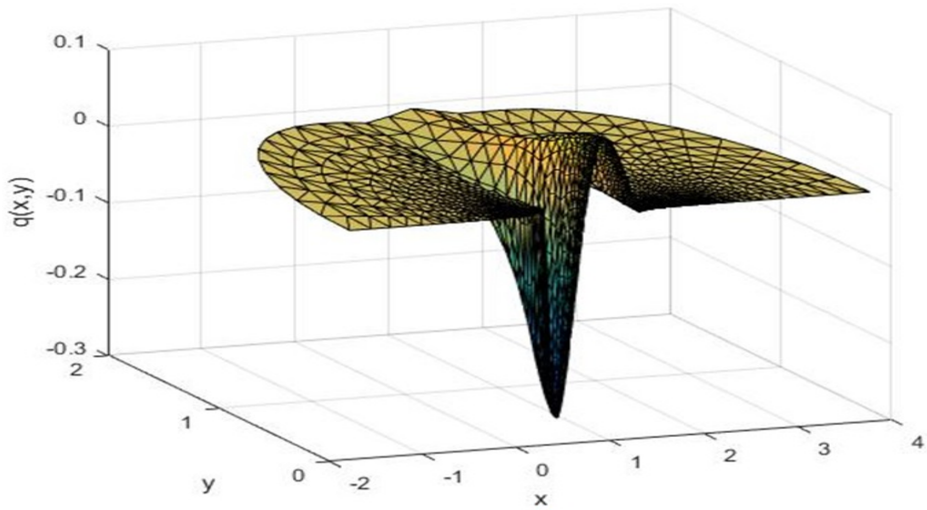


Figure 3. Curves of apparent resistivity in 5, 10, 15 degrees of elevation of relief

The numerical solution of the described method was also tested by comparing with the well-known numerical solution, which was constructed in our earlier works [15] - [16]. We compared the solutions of the integral equation by the method described above and the numerical implementation of the solution of the original integral equation (1) for the surface relief. The best coincidence of the curves of the apparent resistivity of the two programs was achieved with an error of up to 0.5% (Fig. 5), with the length of the measuring line equal to 2 dimensionless units, the angle of inclination of the relief 15 degrees, the number of divisions between the measuring electrodes MN in the case of the method c Fourier transform equal to 4 and when implemented by the direct method equal to 3 (Fig.4a) , Fig.4b)). In the case of applying the direct and inverse Fourier transforms, the total number of cells was 9300, the time for calculation took 1.42 seconds, and the direct method triangulation with the number of nodes 2479 took 12.95 seconds on a personal computer. Although, the use of OpenMP parallelization technology in the latter case allowed speeding up the computations by 2 times [22] .



a)

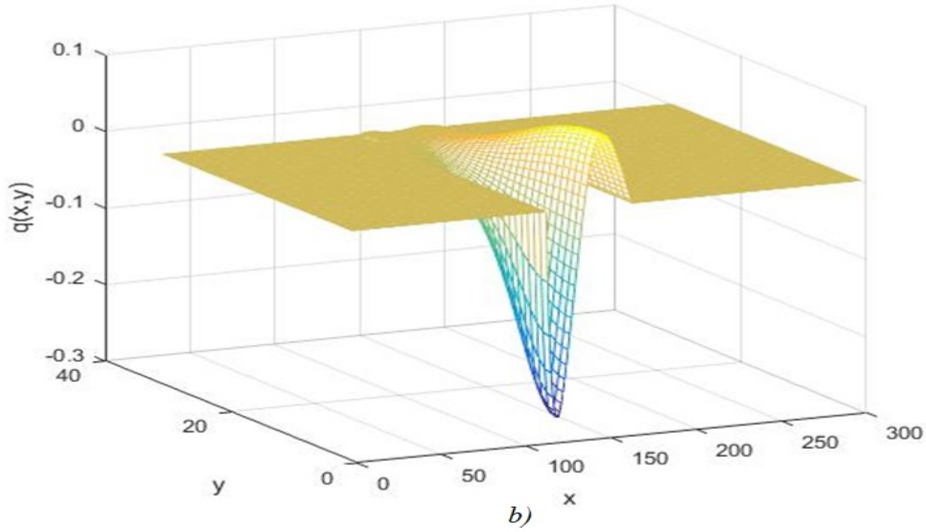


Figure 4. Density distribution of a simple layer $q(M)$ on the surface Γ^0(a) - solution of the initial equation (1) , (b) - by the Fourier transform method

The figure (Fig. 4 a) shows the density distribution of a simple layer q (M) on the surface Γ^0, obtained by a direct method, the grid in this case is constructed as a triangulation that thickens on the measurement line, and the figure (Fig. 4 b) shows the density distribution of a simple layer $q(M)$ on the surface Γ^0 obtained by the method of direct and inverse Fourier transform, the grid in this case is uniform with a logarithmic step Nk = 1024, Ny = 256, the maximum frequency is kymax = 84.

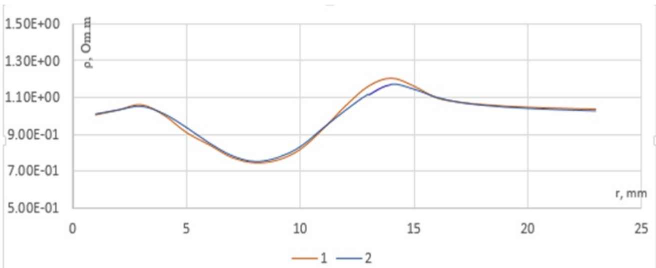


Figure 5. Curves of apparent resistivity (blue — iteration method for equation (1)) (red — Fourier transform)

Calculations have shown that the anomalies of apparent resistivity associated with the relief increase with increasing slope angle of the relief. In the second test, we were able to verify the proximity of the solutions of the two programs, and the calculation time by the method of direct and inverse Fourier transform is significantly less than when solving the original equation (1) head-on, even using the technology of allele pairs on a system with shared memory. Despite the fact that when calculating the inverse Fourier transform, it was necessary to perform numerical integration, the method allowed us to speed up the calculations by reducing the order of integration.

4. Conclusion

This task is two-dimensional in object and three-dimensional in source , the electric field created in the medium is three-dimensional. The method used has shown that it is possible to significantly reduce the time for performing point -source electric field calculations in two-dimensional media and reduce the order of integration. Further researches will be related to the calculation of the addition of contact boundaries and the effect of relief for more complex media models. The ultimate goal of this research - is the modeling of complex structures such as dams and interpretation of false anomalies due to topography.

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