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Generalized $F(R,T)$ cosmological models with fermionic fields

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Abstract. We investigated the gravity model F (R, T), which interacts with a fermion field in a uniform and isotropic flat spacetime FLRW. The main idea and purpose of the work done was to create a mathematical model and find a particular solution for the scale factor a, since it describes the dynamics of the evolution of the Universe. The solutions for this universe are obtained using the Noether symmetry method. With its help, a specific form of the Lagrangian is obtained. And the possible types of the scale factor were found. The evolution of the resulting cosmological model has been investigated.

1. Introduction

The universe is believed to have originated about 14 billion years ago after the Big Bang, when it began to expand rapidly. This period of exponential expansion of the universe is known as the period of initial inflation. Obviously, the inflationary cosmological model is closely related to the theory of gravity. Traditionally, cosmology uses the Einstein-Hilbert gravity model. But he does not describe the initial inflationary stage of the expansion of the Universe. Here it is necessary to use a generalized gravity model containing, for example, the quadratic and higher terms of the Ricci scalar R. As a more general model, a model is often considered that contains not only the curvature scalar, but also such a parameter as the torsion scalar T, as an element of torsional gravity, which also made it possible to give basic theoretical explanations regarding the acceleration of the Universe at a later time. This article discusses the most general type of gravity modification model - the $F(R,T)$ gravity model.

The fermion field is one of the possible reasons for the expansion of the Universe. the description of the evolution of the Universe in scalar field models is well developed. such models describe the accelerated expansion of early and late times. But there are not so many similar works on models with fermionic fields. At the same time, there are no works describing the evolution of the late time with gravity, written by a fermion field in a generalized form from the teleparallel of gravity. In this paper, we will consider cosmology with a gravitational model in its most general form with a fermion field.

2. $F(R,T)$ gravity with fermionic fields
The corresponding action for $F(R,T)$ model interacting with fermions field in FLRW metric is

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} h(u) F(R,T) + Y - V(u) \right\} \tag{1}$$

here $Y = \frac{i}{2}[\overline{\psi}\Gamma^{\mu}\left(\overrightarrow{\partial_{\mu}} - \Omega_{\mu}\right)\psi - \overline{\psi}\left(\overleftarrow{\partial_{\mu}} + \Omega_{\mu}\right)\Gamma^{\mu}\psi]$ – canonical kinetic term of fermion field, $\Gamma^{\nu}_{\mu\lambda}$ – Levi-Civita connection, ψ and its adjugate $\overline{\psi} = \psi^{\dagger}\psi^0$ denote the fermion field, dagger for its complex conjugation, function of time, g is determinant of metric tensor, R is Ricci scalar, T is torsion scalar and $\Omega_{\mu} = -\frac{1}{4}g_{\sigma\nu}\left[\Gamma^{\nu}_{\mu\lambda} - e^{\nu}_{b}\partial_{\mu}e^{b}_{\lambda}\right]\Gamma^{\sigma}\Gamma^{\lambda}$ is spin connection.

In FRLW metric $R_s = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$ and $T_s = 6\frac{\dot{a}^2}{a^2}$, $\dot{a} = \frac{da}{dt}$. Then

$$S = \int d^4 x a^3 \left(\frac{1}{2}h\left(u\right)F\left(R,T\right) - \lambda_1 \left(R - 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - u\right) - \lambda_2 \left(T - 6\frac{\dot{a}^2}{a^2} - v\right) + Y - V\right). \tag{2}$$

To find λ_1 we take from variation with respect to R in the form

$$\lambda_1 = \frac{h}{2}F_R. \tag{3}$$

To find λ_2 we take variation with respect to T:

$$\lambda_2 = \frac{h}{2}F_T. \tag{4}$$

Then action take the form

$$S = \int d^4 x L, \tag{5}$$

were

$$L = \frac{ha^3}{2}\left(F - TF_T - RF_R + uF_R + vF_R\right) + 3ha\dot{a}^2\left(F_T - F_R\right) - 3h\dot{a}a^2\left(\dot{R}F_{RR} + \dot{T}F_{RT}\right) - $$
$$-3h'\left(\dot{\overline{\psi}}\psi + \overline{\psi}\dot{\psi}\right)\dot{a}a^2 F_R + a^3 Y - a^3 V. \tag{6}$$

3. Euler-Lagrange equations
The presence of the Lagrangian makes it possible to use the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\epsilon}} = \frac{\partial L}{\partial \epsilon}, \tag{7}$$

where ϵ is familiar generalized coordinates.

Euler-Lagrange equation for the scale factor a have next form

$$-\frac{F}{2} + \frac{1}{2}\left(TF_T + RF_R\right) - 3H^2(F_T - F_R) + \frac{V}{h} - \frac{Y}{h} + 2\frac{\dot{h}}{h}H\left(F_T - F_R\right) + $$
$$+2H(F_{TT}\dot{T}2\dot{H}\left(F_T - F_R\right) + F_{TR}\dot{R} + F_{TR}\dot{T} + F_{RR}\dot{R} - 2\frac{\dot{h}}{h}\left(F_{RR}\dot{R} + \dot{T}F_{RT}\right) - $$
$$-\ddot{R}F_{RR} - F_{RRR}\dot{R}^2 - 2F_{RRT}\dot{R}\dot{T} - \dot{T}^2 F_{RTT} - \ddot{T}F_{RT} - \frac{\ddot{h}}{h}F_R = 0. \tag{8}$$

For Ricci scalar R:

$$\frac{1}{2}\left(TF_{TR} + RF_{RR}\right) - 3\dot{H}F_{RR} - 6H^2 F_{RR} - 3H^2 F_{TR} = 0. \tag{9}$$

For torsion scalar T:

$$\frac{1}{2}\left(TF_{TT} + RF_{RT}\right) - 3\dot{H}F_{RT} - 6H^2F_{RT} - 3H^2F_{TT} = 0. \tag{10}$$

Energy condition is

$$E_L = 3H^2\left(F_T - F_R\right) - \frac{1}{2}\left(F - TF_T - RF_R\right) - \frac{3\dot{h}}{h}HF_R - 3H\left(F_{RR}\dot{R} + \dot{T}F_{RT}\right) + \frac{V}{h} = 0. \tag{11}$$

4. Noether symmetries

In theoretical physics, the Noether symmetry method is a good way to study models. In particular, it is an excellent approach for correcting cosmological models. The main idea of this method is that if the variational integral remains invariant with respect to a continuous group, then the group generator gives the corresponding conservation law. To do this, consider an infinitesimal generator for this model

$$X = \alpha\frac{\partial}{\partial a} + \beta\frac{\partial}{\partial R} + \gamma\frac{\partial}{\partial T} + \delta\frac{\partial}{\psi} + \epsilon\frac{\partial}{\partial\overline{\psi}} +$$
$$+ \dot{\alpha}\frac{\partial}{\partial\dot{a}} + \dot{\beta}\frac{\partial}{\partial\dot{R}} + \dot{\gamma}\frac{\partial}{\partial\dot{T}} + \dot{\delta}\frac{\partial}{\partial\dot{\psi}} + \dot{\epsilon}\frac{\partial}{\partial\dot{\overline{\psi}}}. \tag{12}$$

Here $\alpha, \beta, \gamma, \delta, \epsilon$ are functions of $a, R, T, \psi, \overline{\psi}$. Time derivatives are:

$$\dot{\alpha} = \alpha_a\dot{a} + \alpha_R\dot{R} + \alpha_T\dot{T} + \alpha_\psi\dot{\psi} + \alpha_{\overline{\psi}}\dot{\overline{\psi}}, \tag{13}$$

$$\dot{\beta} = \beta_a\dot{a} + \beta_R\dot{R} + \beta_T\dot{T} + \beta_\psi\dot{\psi} + \beta_{\overline{\psi}}\dot{\overline{\psi}}, \tag{14}$$

$$\dot{\gamma} = \gamma_a\dot{a} + \gamma_R\dot{R} + \gamma_T\dot{T} + \gamma_\psi\dot{\psi} + \gamma_{\overline{\psi}}\dot{\overline{\psi}}, \tag{15}$$

$$\dot{\delta} = \delta_a\dot{a} + \delta_R\dot{R} + \delta_T\dot{T} + \delta_\psi\dot{\psi} + \delta_{\overline{\psi}}\dot{\overline{\psi}}, \tag{16}$$

$$\dot{\epsilon} = \epsilon_a\dot{a} + \epsilon_R\dot{R} + \epsilon_T\dot{T} + \epsilon_\psi\dot{\psi} + \epsilon_{\overline{\psi}}\dot{\overline{\psi}}. \tag{17}$$

Thus, it is possible to construct the following system of equations

$\dot{a}^2:$ $\alpha 3h(F_T - F_R) + \beta 3ha(F_{TR} - F_{RR}) + \gamma 3ha(F_{TT} - F_{RT}) + \alpha_a 6ha\left(F_T - F_R\right) -$
$\qquad -\beta_a 3ha^2F_{RR} - \gamma_a 3a^2hF_{RT} - \delta_a 3h'\overline{\psi}a^2F_R - \epsilon_a 3h'\psi a^2F_R = 0, \tag{18}$

$\dot{R}^2:$ $3\alpha_R ha^2F_{RR} = 0, \tag{19}$

$\dot{T}^2:$ $3\alpha_T ha^2F_{RT} = 0, \tag{20}$

$\dot{\psi}^2:$ $\alpha_\psi 3h'a^2F_R\overline{\psi} = 0, \tag{21}$

$\dot{\overline{\psi}}^2:$ $\alpha_{\overline{\psi}} 3h'a^2F_R\psi = 0, \tag{22}$

$\dot{a}\dot{R}:$ $-\alpha 6haF_{RR} - \beta 3ha^2F_{RRR} - \gamma 3ha^2F_{RRT} - \alpha_a 3ha^2F_{RR} + \alpha_R 6ha\left(F_T - F_R\right) -$
$\qquad -\beta_R 3ha^2F_{RR} - \gamma_R 3ha^2F_{RT} - \delta_R 3h'\overline{\psi}a^2F_R - \epsilon_R 3h'\psi a^2F_R = 0, \tag{23}$

$\dot{a}\dot{T}:$ $-\alpha 6haF_{RT} - \beta 3ha^2F_{RRT} - \gamma 3ha^2F_{RTT} - \alpha_a 3ha^2F_{RT} + \alpha_T 6ha\left(F_T - F_R\right) -$
$\qquad -\beta_T 3ha^2F_{RR} - \gamma_T 3ha^2F_{RT} - \delta_T 3h'\overline{\psi}a^2F_R - \epsilon_T 3h'\psi a^2F_R = 0, \tag{24}$

$$\dot{a}\dot{\psi}: \qquad -\alpha 6h'\overline{\psi}aF_R - \epsilon 3h'a^2F_R - \gamma 3h'a^2F_{RT}\overline{\psi} - \beta 3h'a^2F_{RR}\overline{\psi} + \alpha_\psi 6ha(F_T - F_R) -$$
$$-\beta_\psi 3ha^2F_{RR} - \gamma_\psi 3ha^2F_{RT} - \delta_\psi 3h'\overline{\psi}a^2F_R - \epsilon_\psi 3h'\psi a^2F_R = 0, \qquad (25)$$

$$\dot{a}\dot{\overline{\psi}}: \qquad -\alpha 6h'\psi aF_R - \delta 3h'a^2F_R - \gamma 3h'a^2F_{RT}\psi - \beta 3h'a^2F_{RR}\psi + \alpha_{\overline{\psi}} 6ha(F_T - F_R) -$$
$$-\beta_{\overline{\psi}} 3ha^2F_{RR} - \gamma_{\overline{\psi}} 3ha^2F_{RT} - \delta_{\overline{\psi}} 3h'\overline{\psi}a^2F_R - \epsilon_{\overline{\psi}} 3h'\psi a^2F_R = 0, \qquad (26)$$

$$\dot{R}\dot{T}: \qquad \alpha_T 3ha^2F_{RR} - \alpha_R 3ha^2F_{RT} = 0, \qquad (27)$$

$$\dot{R}\dot{\psi}: \qquad \alpha_R 3h'\overline{\psi}a^2F_R - \alpha_\psi 3ha^2F_{RR} = 0, \qquad (28)$$

$$\dot{R}\dot{\overline{\psi}}: \qquad \alpha_R 3h'\psi a^2F_R - \alpha_{\overline{\psi}} 3ha^2F_{RR} = 0, \qquad (29)$$

$$\dot{T}\dot{\psi}: \qquad -\alpha_T 3h'\overline{\psi}a^2F_R - \alpha_\psi 3ha^2F_{RT} = 0, \qquad (30)$$

$$\dot{T}\dot{\overline{\psi}}: \qquad -\alpha_T 3h'\psi a^2F_R - \alpha_{\overline{\psi}} 3ha^2F_{RT} = 0. \qquad (31)$$

5. Solution

From $\dot{R}^2$ and $\dot{T}^2$ two possible solutions. First

$$F_{RR} = F_{RT} = 0. \qquad (32)$$

In this case we have presumable answer $F = s_1R + s_2T$, where s_1 and s_2 are functions of $\left(\psi, \overline{\psi}\right)$.

Second possible solution from

$$\alpha_R = \alpha_T = 0. \qquad (33)$$

This possible if α have next form $\alpha = \alpha(a)$. From equations for $\dot{a}\dot{R}$ and $\dot{a}\dot{T}$ we can find new function

$$Z\left(a, \psi, \overline{\psi}\right) = -(2\alpha - \alpha_a a)F_R - \beta haF_R - \gamma haF_{RT}. \qquad (34)$$

So it is possible to exclude β and γ, which allows to get α

$$\alpha = (\alpha_0 a)^n. \qquad (35)$$

All this give next solution for F

$$F = F_0 * (R^m + T^n) + F_{01}, \qquad (36)$$

were F_0 and F_{01} some function of $\psi, \overline{\psi}$, and m, n is constants.

Now some solutions can be found for the scale factor a

$$a = a_{01}(t_01t + t_02)^g, \qquad (37)$$

$$a = a_{02}e^{t_{03}t + t_{04}}, \qquad (38)$$

were $a_{01}, a_{02}, t_{01}, t_{02}, t_{03}, t_{04}, g$ - some constants.

6. Conclusion

In this work, the general model $F(R,T)$ of gravity is investigated, which interacts with a fermion field in a homogeneous and isotropic flat spacetime FLRW. The main idea and purpose of the work done was to create a mathematical model and find a particular solution for the scale factor a, which describes the dynamics of the evolution of the Universe.

For the calculation, the Euler-Lagrange equations and the Noether symmetry method were used. It is important to clarify that the solution gives both power and exponential solutions. What describes the accelerated expansion of the Universe of the initial and present stages of the evolution of the Universe. Here a solution is obtained, which is a consequence of the gravity model taken in the most general form. The result is a solution that satisfies both Einstein's classical theory and Starobinsky's model. It can be concluded that the fermionic field can be considered as a possible impetus to the expansion of the Universe and the observed effects.

7. Acknowledgments

8. References

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