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# Inflation model with viscous fluid

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**Abstract.** We investigated  $f(R)$  gravity with  $k$ -essence using the Starobinsky model as an example, given by the expression  $f(R) = R + \alpha R^2$ . Using the hybrid function of the scale factor, we found the scalar field function and its potential. For the model under consideration, the parameters of the slow roll-off satisfy the inflationary stage. Our model allows us to obtain an accelerated expansion of the Universe during an inflationary period. A non-uniform non-viscous fluid was investigated, and then the viscosity was introduced in the second example. Received fluid equations for the accelerated universe.

## 1. Introduction

Inflation was introduced for solution number of cosmological problems such as horizon problems, entropy problems, monopole problems, flatness problems arising from the investigation of the very early universe [1]-[4]. There are several models for instance the Starobinsky model, the chaotic inflation model which attempt to solve that problem in sources. Among that models one of the most important is the Starobinsky model [5]. In this article we investigation one of the simple modification general relativity known as the Starobinsky model is given by the expression  $f(R) = R + \alpha R^2$ .

Models involving inhomogeneous viscous fluids began to be investigated after the discovery of cosmic acceleration at the end of the 20th century and the appearance of the dark energy problem, in the Friedman-Robertson-Walker (FRU) universe. Cosmological observation data do not except the most complex models dark energy compared to the simplest one obtained by introducing the cosmological constant into the Einstein equation. That models dark energy have corresponding representation in fluid-like form. Models  $f(R)$  gravity can also be written in this form, as a result, using the framework of general relativity, it is simplified and easier to analyze [6].

$k$ -essence model was proposed as a generalization of the scalar field. Historically, this model was first proposed to describe the era of inflation [7], and later to describe dark energy [8]-[11]. We choose the measurement units so that.



## 2. $f(R)$ model of gravity with $k$ - essence

Lets consider action  $f(R)$  gravity and  $k$ -essence

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [f(R) + 2K(X, \varphi)], \quad (1)$$

where  $g$  is determinant of the metric tensor  $g_{\mu\nu}$ ,  $K$  is function of its arguments and  $\varphi$  scalar function. Here

$$X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi \quad (2)$$

is the canonical kinetic term of the scalar field.  $\nabla_\mu$  covariant derivative. The most general space-time metric consistent with the cosmological principle is the metric FRW. The interval for the FRW metric is as follows [12]-[14]

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (3)$$

where  $a$  - scale factor depends on  $t$ . For the action (1) together with metric FRW (2) the Lagrangian takes the form

$$L = fa^3 - f_R Ra^3 - 6f_{RR} \dot{R} a^2 - 6f_R \dot{a}^2 a + 2a^3 K. \quad (4)$$

Using the Euler-Lagrange equation and the zero-energy condition, we obtained a complete system of equations of motion for the considered model

$$3H^2 = \rho, \quad (5)$$

$$3H^2 + 2\dot{H} = -p, \quad (6)$$

$$K_X \ddot{\varphi} + \dot{\varphi} (\dot{K}_X + 3HK_X) - K_\varphi = 0, \quad (7)$$

$$\dot{\rho} + 3H(p + \rho) = 0, \quad (8)$$

where

$$\rho = \frac{1}{f_R} \left( -3f_{RR} \dot{R} H + \frac{1}{2} f_{RR} - \frac{1}{2} f + 2K_X X - K \right), \quad (9)$$

$$p = \frac{1}{f_R} \left( f_{RRR} \dot{R}^2 + f_{RR} (\ddot{R} + 2\dot{R} H) - \frac{1}{2} f_{RR} R + \frac{1}{2} f + K \right). \quad (10)$$

Equations (5), (6) are Friedman equation, (7) is Klein-Gordon equation and (8) – conservation equation. The dot above the symbol means time derivative  $t$ .

We choose the  $k$ -essence Lagrangian in the form  $K = X - V(\varphi)$ , for to find solutions to the system of equations, where the kinetic term of the scalar field  $X$  (2) for the FRW metric (3) is equal to

$$X = \frac{1}{2} \dot{\varphi}^2.$$

We use Starobinsky's model  $f(R) = R + \alpha R^2$ , also called quadratic gravity [5], to find a solution

$$f_R = 1 + 2\alpha R, \quad f_{RR} = 2\alpha, \quad f_{RRR} = 0. \quad (11)$$

Let us write down the equations of motion (5)-(10) taking into account (1)

$$3H^2 = \rho, \quad (12)$$

$$3H^2 + 2\dot{H} = -p, \quad (13)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{\varphi} = 0, \quad (14)$$

$$\dot{\rho} + 3H(p + \rho) = 0, \quad (15)$$

where

$$\rho = \frac{1}{1 + 2\alpha R} \left( -6\alpha\dot{R}H + \frac{1}{2}\alpha R^2 + \frac{1}{2}\dot{\varphi}^2 + V \right), \quad (16)$$

$$p = \frac{1}{1 + 2\alpha R} \left( 2\alpha\ddot{R} + 4\alpha\dot{R}H - \frac{1}{2}\alpha R^2 + \frac{1}{2}f + \frac{1}{2}\dot{\varphi}^2 - V \right). \quad (17)$$

In order to find the function of the scalar field  $\varphi$ , we add in pairs the equations (12), (13) and (16), (17) and equate the results. We get

$$\dot{\varphi}^2 = -2\dot{H}(1 + 2\alpha R) - 2\alpha(\ddot{R} - \dot{R}H). \quad (18)$$

Equation (18) can be rewritten in terms of the Hubble parameter  $H$

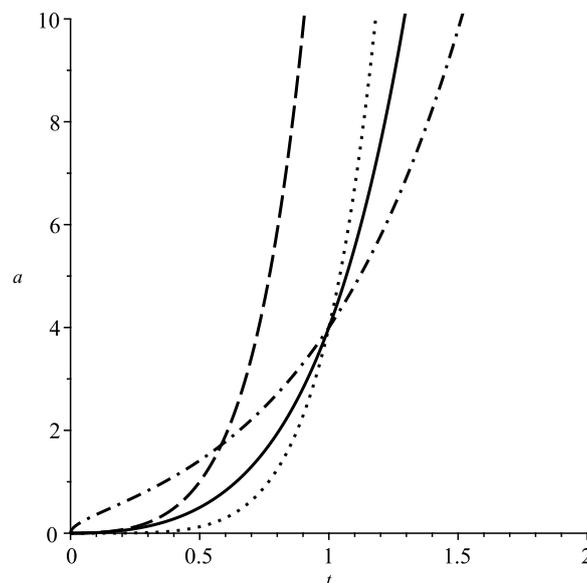
$$\dot{\varphi}^2 = -12\alpha(H^{(3)} + 3H\ddot{H} + 6\dot{H}^2) - 2\dot{H}. \quad (19)$$

### 3. Finding a solution

We choose a scale factor in the form of a hybrid function to find a solution to the system of equations of motion (12)-(17)

$$a = a_0^{\gamma t} t^{\beta}, \quad (20)$$

where  $a_0, \gamma, \beta$  some constants.



**Figure 1.** Scale factor  $a$  via time  $t$

Figure 1 illustrates dependence of the scale factor  $a$  (20) via time  $t$  for  $a_0 = 2, \gamma = 2, \beta = 2$  (solid line),  $a_0 = 2, \gamma = 4, \beta = 2$  (dotted line),  $a_0 = 2, \gamma = 2, \beta = 4$  (pointed line),

$a_0 = 2, \gamma = 2, \beta = \frac{1}{2}$  (dash-dotted line). The scale factor has the meaning of the radius of the universe. In order for our model to describe the accelerated expansion of the universe, it is necessary that all constants included in expression (20) are greater than zero.

For further search for a solution, we apply the following expressions are useful

$$\begin{aligned} \dot{a} &= \alpha \ln a_0 a_0^{\gamma t} t^{\beta} + \beta a_0^{\gamma t} t^{\beta-1}, \quad H = \frac{\dot{a}}{a} = \gamma \ln a_0 + \frac{\beta}{t}, \\ \dot{H} &= -\frac{\beta}{t^2}, \quad \ddot{H} = \frac{2\beta}{t^3}, \quad H^{(3)} = -\frac{6\beta}{t^4}. \end{aligned} \quad (21)$$

Substituting expressions (21) into (19), we obtain the value of the function of the square of the differential of the scalar field

$$\dot{\varphi}^2 = \frac{2\beta}{t^2} - \frac{72\alpha\beta\gamma \ln a_0}{t^3} + \frac{72\alpha\beta(1-2\beta)}{t^4}. \quad (22)$$

Knowing the form of the scalar field function (22) from the Klein-Gordon equation (14), we find the scalar field potential

$$\begin{aligned} V &= \frac{\beta(3\beta-1)}{t^2} + \frac{18\alpha\beta(1-2\beta)(3\beta-2)}{t^4} + \\ &+ 3\gamma \ln a_0 \left[ \frac{2\beta}{t} + \frac{4\alpha\beta(9-14\beta)}{t^3} - \frac{36\alpha\beta\gamma \ln a_0}{t^2} \right] + V_0, \end{aligned} \quad (23)$$

where  $V_0$  integration constant.

#### 4. Slow Roll parameters

The slope of the potential  $\varepsilon(\varphi)$  and the curvature  $\eta(\varphi)$ , which are called the parameters of the slow roll are defined through the potential and the scalar field function, as follows [15]

$$\varepsilon(\varphi) = \frac{1}{2} \left( \frac{V_{\varphi}}{V} \right)^2, \quad \eta(\varphi) = \frac{V_{\phi\phi}}{V}. \quad (24)$$

If the potential of the scalar field and the function of the scalar field are expressed in terms of time  $t$ , then expressions (24) can be rewritten as [16]-[17]

$$\varepsilon(t) = \frac{1}{2\dot{\varphi}^2} \left( \frac{\dot{V}}{V} \right)^2, \quad (25)$$

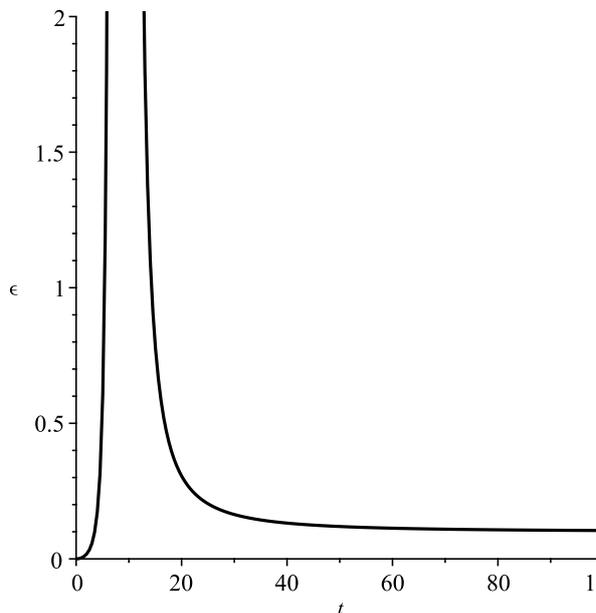
$$\eta(t) = \frac{\ddot{V}}{\dot{\varphi}^2} - \frac{\dot{V}\ddot{\varphi}}{\dot{\varphi}^3}. \quad (26)$$

For the emergence and continuation of the inflationary stage, it is necessary that these parameters are in the area

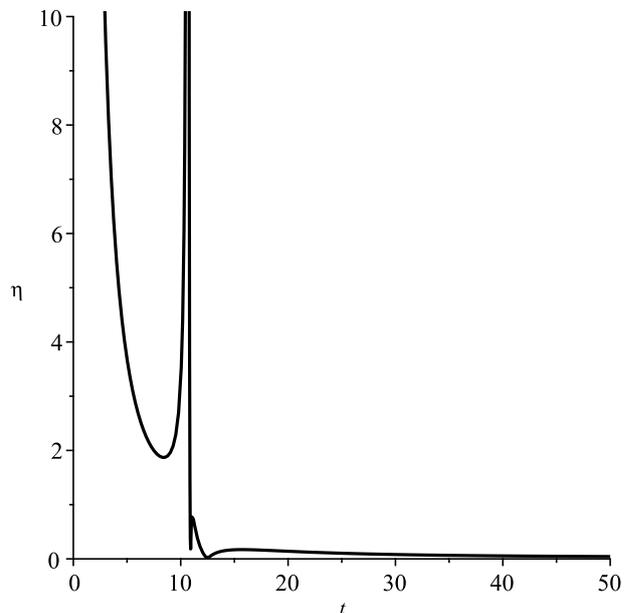
$$\varepsilon(\phi) \ll 1, \quad (27)$$

$$|\eta(\phi)| \ll 1. \quad (28)$$

As can be seen from Figures 2 and 3, the slow roll parameters our model for the potential of the scalar field (23) and the function of the scalar field (22) satisfy this condition. When the slow roll parameters tend to 1, the inflationary stage is exited.



**Figure 2.** Potential slope  $\varepsilon(t)$ , at  $\alpha = \frac{1}{6}, \beta = 10, \gamma = 2, a_0 = 2$ .



**Figure 3.** Curvature of potential  $\eta(t)$  at  $\alpha = \frac{1}{6}, \beta = 10, \gamma = 2, a_0 = 2$ .

### 5. Hybrid Scale Factor Fluids

Pressure  $p$  and energy density  $\rho$  included in the Friedman equations (12)-(13) must satisfy the conservation law. We investigate the general form of the equation of state for an inhomogeneous viscous fluid for our model [18]-[22]

$$p = \omega(\rho)\rho - B(a(t), H, \dot{H}, \dots), \quad (29)$$

where parameters of equation of state  $\omega(\rho)$  may depend on the energy density, and the bulk viscosity  $B(a(t), H, \dot{H}, \dots)$  is a function of its arguments – scale factor  $a(t)$ , Hubble parameter  $H$  and its derivatives. To fulfill the thermodynamic law of increasing entropy, the bulk viscosity must be positive [18], [23]-[24]. The energy-momentum tensor of a fluid  $T_{\mu\nu}$  has the form

$$T_{\mu\nu} = \rho u_\mu u_\nu + [\omega(\rho)\rho + B(a(t), H, \dot{H}, \dots)](g_{\mu\nu} + u_\mu u_\nu), \quad (30)$$

where  $u_\mu = (1, 0, 0, 0)$  4-velocity vector. The energy conservation law for a fluid is generally written as

$$\dot{\rho} + 3H\rho(1 + \omega(\rho)) = 3HB(a(t), H, \dot{H}, \dots). \quad (31)$$

Fluids that, in the modern era, satisfy realistic cosmological models, but describe a different evolution compared to the case of the cosmological constant are interested for us. The effective parameter of the equation of state for fluid (29) has the form

$$\omega_{eff} = \frac{p}{\rho} = \omega(\rho) - \frac{B(a(t), H, \dot{H}, \dots)}{\rho}. \quad (32)$$

Modern cosmological observational data show that  $\omega_{eff} \simeq -1$  [25]. For the case of ideal fluids,  $\omega$  should be very close to  $-1$ , but different values are allowed for different non-ideal fluid

models [6]. From the Friedman equation (29) for our model, we find the value of the density expression  $\rho$

$$\rho = 3 \left( \gamma \ln a_0 + \frac{\beta}{t} \right)^2. \quad (33)$$

Find the time derivative of the energy density  $t$

$$\dot{\rho} = -\frac{6\beta}{t^2} \left( \gamma \ln a_0 + \frac{\beta}{t} \right). \quad (34)$$

Now let's analyze fluids using conservation law (15). To do this, consider the general form of the equation of state (29) for this type of fluids. First, we investigate a non-uniform non-viscous fluid, and then we introduce viscosity. In the case of an inviscid fluid  $B(a(t), H, \dot{H}, \dots) = 0$ . From the conservation law we obtain

$$p = -\rho + \frac{2}{\beta} \left[ \left( \frac{\rho}{3} \right)^{\frac{1}{2}} - \gamma \ln a_0 \right]^2 \quad (35)$$

and the parameter of the equation of state is

$$\omega = -1 + \frac{2}{\beta\rho} \left[ \left( \frac{\rho}{3} \right)^{\frac{1}{2}} - \gamma \ln a_0 \right]^2. \quad (36)$$

We can also write the equation for a fluid using other forms of the equation of state and introducing the bulk viscosity. Consider the case where the constant parameter of the equation of state is  $\omega = -1$  and the bulk viscosity depends only on the Hubble parameter

$$B(a(t), H, \dot{H}, \dots) = 3H\zeta(H), \quad (37)$$

where  $\zeta(H) > 0$  bulk viscosity. In this case, the equation of state of the fluid is

$$p = -\rho - \frac{2}{\beta}(H - \gamma \ln a_0)^2, \quad (38)$$

$$\zeta(H) = \frac{2}{3H\beta}(H - \gamma \ln a_0)^2. \quad (39)$$

## 6. Conclusion

We investigated the  $k$ -essence and  $f(R)$  gravity model. Starobinsky model  $f(R) = R + \alpha R^2$  was used for search solution. Choosing a hybrid solution for the scale factor, we found the scalar field function and its potential. For the model under consideration, the slow roll parameters of the satisfy the inflationary stage. Our model allows us to obtain an accelerated expansion of the universe during an inflationary period. With the passage of time, the field decreases, a slow roll occurs, the viscosity has less effect, and the universe leaves the inflationary regime, which is shown by the hybrid dynamics of the change in the law of the universe's expansion.

The study of inhomogeneous viscous fluids in the Friedman-Robertson-Walker universe is important. Inhomogeneous viscous fluids have a general equation of state form that can be used to describe the current era of dark energy or the early era of inflation. Modified  $f(R)$  gravity theories have a corresponding fluid representation. During the contraction phase, the energy density increases and decreases in the expanding universe, but there are some areas where this behavior is opposite. We analyzed fluids using the conservation law, considering the general form of the equation of state for this type of fluids. In the first example, a non-uniform non-viscous fluid was examined, and then in the second example, the viscosity was introduced. We have obtained fluid equations for an accelerated Universe that violate the strong energy condition: these fluid models can be free of temporary singularities or can admit their presence depending on the value of the equation of state parameter and the bulk viscosity coefficients.

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### References

- [1] Guth A H 1981 *Phys. Rev. D* **23** 347
- [2] Linde A D 1982 *Phys. Lett. B* **108** 389
- [3] Linde A D 2008 *Lect. Notes Phys* **738** 1
- [4] Gorbunov D S and Rubakov V A 2011 *Introduction to the theory of the early universe: Cosmological perturbations and inflationary theory* (USA: World Scientific)
- [5] Starobinsky A A 1980 *Phys. Lett. B* **91** 99
- [6] Myrzakul S, Myrzakulov R, Sebastiani L 2014 *Astrophys.Space Sci.* **350** 845-853
- [7] Armendariz-Picon C, Damour T, Mukhanov V F 1999 *Phys. Lett. B* **458** 209-218
- [8] Armendariz-Picon C, Mukhanov V F, Steinhardt P J 2010 *Phys. Rev. D* **63** 3510
- [9] Armendariz-Picon C, Mukhanov V F, Steinhardt P J 2000 *Phys. Rev. Lett* **85** 4438-4441
- [10] Chiba T, Okabe T, Yamaguchi M 2000 *Phys. Rev. D* **62** 3511
- [11] De Putter R, Linder E V 2007 *Astropart. Phys.* **28** 263-272
- [12] Razina O, Tsyba P, Meirbekov B, Myrzakulov R 2019 *Int. J. Mod. Phys. D* **28** 1950126
- [13] Razina O, Tsyba P and Sagidullayeva Z 2019 *Bull. Univ. Karaganda Phys.* **425** 94
- [14] Razina O V, Tsyba P Yu 2018 *Bull. Eurasian National Univ. Phys. Astr.* **3** 33-40
- [15] Liddle A, Lyth D 2000 *Cosmological Inflation and Large-Scale Structure* (England: Cambridge university press)
- [16] Tsyba P, Razina O, Barkova Z, Bekov S and Myrzakulov R 2019 *J. Phys. Conf. Ser* **1391** 012162
- [17] Razina O V, Tsyba P Yu, Myrzakulov R, Meirbekov B, Shanina Z 2019 *J. Phys. Conf. Ser* **1391** 012164
- [18] Myrzakulov R, Sebastiani L 2014 *Astrophys.Space Sci.* **352** 281-288
- [19] Capozziello S, Cardone V. F, Elizalde E, Nojiri S and Odintsov S D 2006 *Phys. Rev. D* **73** 043512
- [20] Nojiri S and Odintsov S D 2005 *Phys. Rev. D* **72** 023003
- [21] Nojiri S and Odintsov S D 2006 *Phys. Lett. B* **639** 144
- [22] Nojiri S, Odintsov S D, Oikonomou V K 2017 *Phys. Rep.* **692** 1-104
- [23] Brevik I H, Gorbunova O 2005 *Gen.Rel.Grav.* **37** 2039-2045
- [24] Brevik I H, Gorbunova O and Shaido Y A 2005 *Int. J. Mod. Phys. D* **14** 1899
- [25] Planck Collaboration, Akrami Y. et al. 2020 *A&A* **641** 61