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INVESTIGATION OF THE ENERGY DEPENDENCE OF THE INTERACTION POTENTIALS OF THE $^{16}O+^{12}C$ NUCLEAR SYSTEM WITH A SEMI-MICROSCOPIC METHOD

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The study of the collision of heavy ions with light nuclei at low energy is important in nuclear physics, thermonuclear energy, and astrophysics. The high-precision values of the nuclear system described at an energy close to the Coulomb barrier are used to control the synthesis of light nuclei inside thermosynthesis. For cross sections of reactions of light nuclei on the sun, plasma, and stars, we can use the parameters we have set. The article presents a microscopic approach to describing the process of nuclear-nuclear interaction. In the phenomenological approach, which determines empirical values based on comparison with experimental data, it is possible to find many sets of parameters with potentials that characterize the cross-section well. But the question arises which of them are real. Therefore, it is necessary to additionally describe microscopic potentials. For the same reason, a semi-microscopic analysis was carried out, which describes the imaginary part of the nuclear potential on the basis of an optical model, and the real part on a double-folding model. The folding potential is constructed depending on the effective nucleon-nucleon interaction based on the matrix element of two nucleons and the density of the nucleon distribution. As a result of the analysis, differential cross sections and optimal parameters were determined that well characterize the experimental cross sections of the $^{16}O+^{12}C$ *nuclear system at energies* E_{Lab}*=20, 24, 36 MeV. The coefficients of normalization of differential cross sections, described on the basis of real microfolding potentials, were determined in the range N=0.85-1.0.*

***Keywords**:* elastic scattering, folding model, matter density distribution, nucleon-nucleon interaction.

Introduction

Microscopic studies of the structural characteristics and mechanism of nuclear matter give their results. Based on the analysis of the scattering of composite particles in the nucleus at lower energy, it is possible to obtain important information about the properties of the internuclear potential [1]. To obtain information about the structure of nuclei and nuclear matter, it is necessary to know the mechanism of their interaction. To do this, it is necessary to construct nuclear-nuclear optical potentials and, on this basis, simulate the observed interaction characteristics.

As for the distribution of the density of nuclei, one of the fundamental achievements of recent times in this field is the discovery of expanded "tails" (halos) in the distribution of neutron and proton densities of light radioactive nuclei [2]. A detailed study of the differential cross-sections of elastic scattering of alpha particles and p-shell nuclei with nuclei led to the discovery of the phenomenon of "nuclear rainbow. In turn, it became necessary to introduce effective nucleon-nucleon forces depending both on the density of the nuclear medium in the area of mutual penetration of nuclei and on the collision energy of nuclei [3, 4]. The DF (Double-folding) model calculates the nuclear-nuclear potential using density-dependent of the 3 parameters of Michigan Yukawa (M3Y) interactions describing the density and energy dependence of the optical potential of the nucleon. Effective interaction is obtained by density-dependent forms by comparing the three elements selected in the Yukawa sum matrix. This model can describe the structure of the nucleus, since it is studied on the basis of summation of the central part of the nucleon-nucleon (NN) interaction with the ground states of the density of the beam ion and the target nucleus.

The article describes experimental elastic scattering cross-sections of the $^{16}O+^{12}C$ reaction with differential cross sections based on the standard optical model (OM). In addition, the DF model defines differential cross sections in which nuclear-nuclear potentials are constructed with effective forces NN.

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Since the elastic scattering data clearly depend on the density of the distribution of matter in the nucleus, it is necessary to analyze the interaction of M3Y. The introduction of a real density dependence leads to a significant change in both the strength and the folding potential. The dependence on the same density is called the DDM3Y interaction [5]. In order to obtain the real part of micro folding potentials, density-dependent CDM3Y, BDM3Y DDM3Y types of effective M3Y interactions were used.

The study of the process of nuclear-nuclear interaction on the basis of theoretical ideas about the effective interaction of two nucleons gives reliable assumptions about the structure of the nucleus. The effective NN-interaction of m3u is carried out on the basis of the proton-neutron interaction matrix and the density distribution integral of nucleons. Microscopic analysis explicitly takes into account correlations based on effective NN forces to clarify the shape of the real part of the nuclear potential.

1. Construction of NN interaction potentials

The distribution of nuclear matter is constructed in a factorized form depending on coordinates and density in an effective interaction of the M3Y type [5]. The interaction potential of the beam nucleus and the target nucleus depends on the E/A_1 - nucleon energy and the density of the nuclei in the region of their overlap [6]. Equation of dependence of the optical potential of a nucleon on density and energy,

$$\upsilon_{D(EX)}(E,\rho,s) = g(E)\,f(\rho)\,\upsilon'_{D(EX)}(s), \tag{1}$$

where, $\upsilon_{D(EX)}$ - Direct (D) and Exchange (EX) potentials, $g(E)$ - dependence of potential on energy, $f(\rho)$ - density dependence function.

Effective NN potentials are determined by the sum of M3Y potentials in the region of interacting nuclei. Isoscalar and isovector M3Y potentials are parametrized as follows [6].

$$\vartheta_{0n}^{D(EX)} = \sum_{i=1,2,3} N_i \frac{e^{-\mu_j r}}{\mu_j r} \quad . \tag{2}$$

Table 1. M3Y-Reid, M3Y-Paris N$_i$, μ_i parameters, and coefficients of potentials [7, 8]

N_i	i	N_1	N_2	N_3
Potential	μ_i, fm^{-1}	4.0	2.5	0.7072
M3Y-Reid	ϑ_{00}^{D}, MeV ϑ_{00}^{EX}, MeV	7999.0 4631.375	-2134.25 -1787.125	0 -7.8474
M3Y-Paris	ϑ_{00}^{D}, MeV ϑ_{00}^{EX}, MeV	11061.625 -1524.25	-2537.5 -518.75	0 -7.8474

At lower energies, it is important to clearly consider the effects of exchange. Therefore, effective NN takes into account direct, exchange components of interaction [8] and even, odd components of central forces [9]:

$$\upsilon_D(s) = 7999{,}0\frac{e^{-4s}}{4s} - 2134{,}25\frac{e^{-2{,}5s}}{2{,}5s}, \tag{3}$$

$$\upsilon_{EX}(s) = 4631{,}4\frac{e^{-4s}}{4s} - 1787{,}1\frac{e^{-2{,}5s}}{2{,}5s} - 7{,}8474\frac{e^{-0{,}7072s}}{0{,}7072s}, \tag{4}$$

$\upsilon_D(s)$- the fraction of the third term of the direct potential is zero.

Direct and exchange component of effective M3Y interaction based on the G-matrix element of the Paris potential [17]:

$$\upsilon_D(s) = 11061{,}6\frac{e^{-4s}}{4s} - 2537{,}5\frac{e^{-2{,}5s}}{2{,}5s}, \quad (5)$$

$$\upsilon_{EX}(s) = -1524{,}0\frac{e^{-4s}}{4s} - 518{,}8\frac{e^{-2{,}5s}}{2{,}5s} - 7{,}8474\frac{e^{-0{,}7072s}}{0{,}7072s}, \quad (6)$$

The direct part of the potential is completely elastic, only at the expense of the exchange component is absorption.

2. Construction of the density dependence function

Harmonic oscillator (HO) and Three-parameter Fermi - (3pF) models were used in the distribution of the density of the nuclei of the ^{16}O-beam and oh, ^{12}C-targets [12, 13]. The general form of the density dependence factor and the correlation function is determined by the following equation [14, 17]:

$$f(\rho) = C_{\rho}^{'}(E)(1 + \alpha(E)e^{-\beta(E)(\rho_1+\rho_2)}), \quad (7)$$

M3Y- Paris energy-dependent form of potential,

$$g(E) = (1 - 0.003E/A), \quad (8)$$

dependent density type CDM3Y:

$$f(\rho) = C_{\rho}^{'}(E)(1 + \alpha(E)e^{-\beta(E)(\rho_1+\rho_2)} - \gamma\rho(r)), \quad (9)$$

CDM3Yn (n=1) C_ρ, α, β, γ the parameters are given in Table 2 [15].
M3Y-Reid [2, 8, 9] energy-dependent type of potential:

$$g(E) = (1 - 0.002E/A), \quad (10)$$

M3Y-Reid for the potential, we use the form of density dependence in DM3Y [6].

$$f(\rho) = C(1 - \gamma\rho(r)), \quad (11)$$

BDM3Y1- Reid, C_ρ, γ - the parameters are given in Table 2.

DDM3Y0 The formula of the energy dependence of the potential in the d-independent form of interaction $g(E) = 276(1 - 0.005E/A)$ is applied.

Table 2. M3Y interaction coefficients depending on the density

Density-dependent version	C_ρ	a	β	γ
CDM3Y1-Paris	0.3429	3.0232	3.5512	0.5
BDM3Y1-Reid	1.2521	0.0	0.0	1.7452
DDM3Y0-Reid	1.0	0.0	0.0	0.0

The table shows n=1 for CDM3Yn, BDM3Yn interactions. In the article by Khoa Dao Tien [10, 18, 19], the dependence of the scattering density of the ^{16}O ion on the ^{12}C nucleus on CDM3Y-Paris, BDM3Y-Reid was analyzed. From here, the K-incompressibility parameter is selected, which is in good agreement with experimental values in the range of 150-210.

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3. Discussions and results

Nuclear micro folding potentials for the ^{12}C+^{16}O nuclear system are calculated in the C++ program for energy of 20, 24, 35 MeV. M3Y-Reid, M3Y-Paris when calculating the potentials , the values of the density of nuclear matter were calculated using the formula [11].

$$\rho_0 = A \Big/ (4\pi a^3 \sqrt{\pi}\left[\frac{1}{4}+\frac{3}{8}\alpha\right]), \tag{12}$$

For the core of the ^{16}O beam, the values ρ_0 =0.17 fm^3 are assumed, for the core of the ^{12}C target ρ_0 =0.16 fm^3. As a result of the calculation, folding potentials were obtained depending on the density of CDM3Y1, BDM3Y1, DDM3Y0 at energies of 20, 24, 35 MeV for a nuclear system of ^{16}O+^{12}C. We use this folding potential instead of the real part of the OM potential:

$$U(r) = NV_F(r) - iW_0 f\left(r, r_W, a_W\right) + V_C(r), \tag{13}$$

where, N - the rationing factor, V_F - folding potentials,W_0 - imaginary potential, r_W, a_W - diffusion and radius of the imaginary potential, $V_C(r)$ - Coulomb potential.

The folding potential in this formula:

$$V_F = \iint \rho_1(r_1)\rho_2(r_2)\vartheta_{NN}(s)d^3r_1 d^3r_2, \quad s = r + r_2 - r_1, \tag{14}$$

where ϑ_{NN} - effective interaction potential NN, $\rho_1(r_1)$ and ρ_2 (r_2) the distribution of the density of matter between the flying particle and the target nuclei, respectively.

(13) the real part of the optical potential in the localized folding potential formula is estimated within the density form. The parameters of the Woods-Saxon potential, obtained as an imaginary part of the potential, are determined by the Fresco program. Thus, based on the optical model, we obtain differential cross sections semi-microscopically. Figure 1-3.

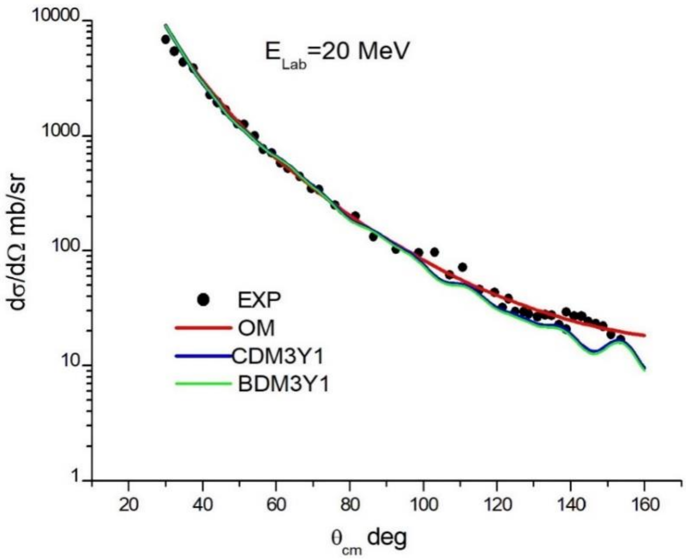

Fig.1. Cross sections of the ^{16}O+^{12}C system of OM and DFM at energy E_{Lab} = 20 MeV.

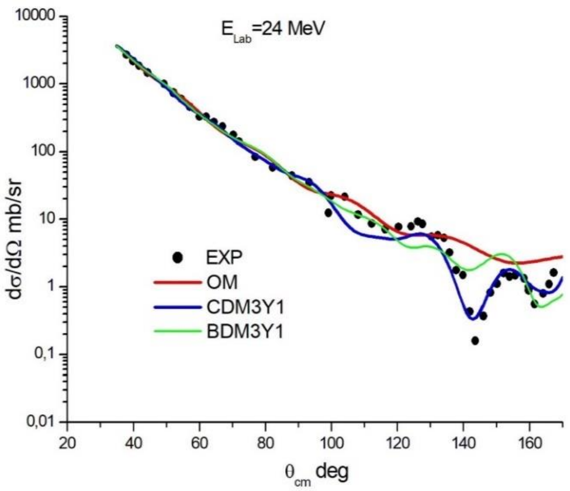

Fig.2. Cross sections of the $^{16}O+^{12}C$ system of OM and DFM at energy E_{Lab} = 24 MeV.

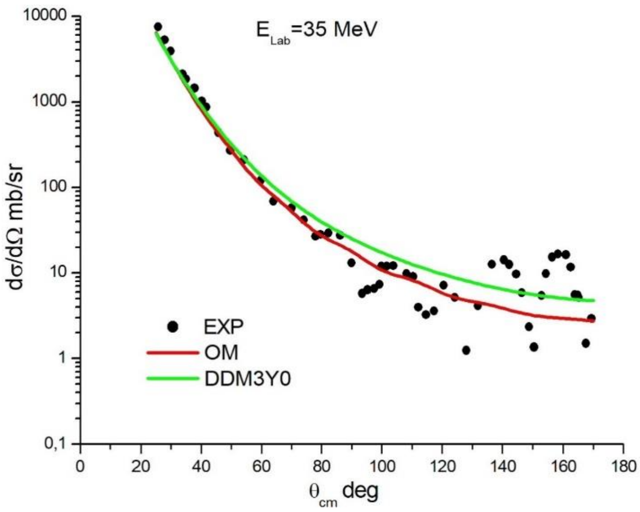

Fig.3. Cross sections of the $^{16}O+^{12}C$ system of OM and DFM at energy E_{Lab} = 35 MeV.

Differential cross sections depending on the determined density CDM3Y1, BDM3Y1, DDM3Y0 describe the experimental cross-section at a full angle. Experimental data in the studied nuclear system $^{16}O+^{12}C$ used the energies lab =20, 24, 35 MeV from the literature [16].

The following set of parameters describing the experimental data was determined: V_0 - real potential, W_0 - imaginary potential, a_r, a_w - diffuse of real, imaginary potential, Nr - coefficient of normalization, χ^2/N - error, Jv, Jw - integrals of the real and the imaginary potentials and σ - cross section (Table 3). The peculiarity of the analysis is that when searching for potentials, the radii of real, imaginary parts and the radii of coulombs were fixed.

Table 3. Parameters of the $^{16}O+^{12}C$ system at an energy of 20, 24, 35 MeV, DFM CDM3Y1, BDM3Y1, DDM3Y0, fix: r_r=1.2 fm, r_w=1,25 fm. r_C=1.3 fm.

E_{Lab}, MeV	Potential	V_0, MeV	a_r, fm	W_0, MeV	a_W, fm	N_r	χ^2/N	σ	J_V	J_W
20	OM	103.0	0.35	5.0	0.9		8.4	212	447.6	28.94
	CDM3Y1			5,0	0.9	0.9	4.0			
	BDM3Y1			5.0	0.9	0.9	2.5			
24	OM	105.0	0.465	5.97	0.407		1,4	476	468.52	29.58
	CDM3Y1			5,97	0,407	0.85	4.6			
	BDM3Y1			5.97	0.407	1.0	4.9			
35	OM	80.0	0.233	9.8	0.2		5.2	938	411.51	28.49
	DDM3Y0			9.8	0.2	0.8	7.2			

Conclusion

The folding potentials of M3Y-Read and M3Y-Paris effective NN interactions are determined. A semimicroscopic analysis of the $^{16}O+^{12}C$ nuclear system based on OM using core-core folding potentials has been carried out. The differential elastic scattering cross sections at energies E_{lab}=20, 24, 35 MeV were chosen for analysis within the framework of the optical model and the folding model (CDM3Y1, BDM3Y1, DDM3Y0). The variants BDM3Y1-Raid, CDM3Y1-Paris and DDB3Y1-Fatal are described only by the front (up to 90^0) angles, applicable to the experimental data of other authors [20].

In our work, up to 180^0 corners are described. The coefficient of N_r - normalization of differential cross sections constructed using folding potentials was found in the range of 0.8-1.0 (Table 3). The presence of an imaginary part of the potentials at a low energy of less than 10 MeV indicates the transparency (elasticity) of the optical potential. The decrease in the real part and the increase in the imaginary part of the potentials from the energy dependence in laboratory energies at 20-35 MeV retains a global pattern. In accordance with the energy dependence of the real and imaginary potentials, the volume integral changes monotonically. It is important to solve the energy problem by applying the values of high-precision parameters determined at low energy to the synthesis of light nuclei inside thermosynthesis. When studying the cross section of the reaction of light nuclei on the sun, plasma and stars, it is possible to use the potentials (other parameters) of nuclear systems that we have determined.

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