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Mathematical foundations of modeling economic indicators

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Abstract. This paper describes the mathematical foundations for modeling multidimensional economic indicators that form a linear space with mathematical operations defined therein, such as addition and multiplication. The concept of factors is introduced as an enumerated type of possible values, as well as the concept of indicators that can depend on factors and have a multidimensional structure. We determined the space of factors and the space of measurements in which the basic arithmetic operations are identified and their properties are proved. Examples of modeling real economic indicators are given on the example of calculating the cost of services for the transportation of goods by rail. An important advantage of using the described abstract mathematical space of meters is the ability to automatically track the integrality or differentiability of economic indicators in the corresponding economic information systems. The proposed mathematical apparatus can be used to solve various economic problems and to design the structure of an economic information systems' database. References are given to the results of specific projects carried out in the Republic of Kazakhstan, in which the proposed mathematical apparatus is used.

1. Introduction

Economic indicators have a multidimensional structure by their nature, and it is determined by the decomposition of the values of these indicators according to various criteria. For example, an indicator such as "population size" depends on various characteristics such as gender, education, etc. Thereby, according to these features, the indicator "population size" has subsidiary indicators such as "number of men", "number of women", "population with higher education", "population with secondary education", "population with primary education" etc. It is much easier and more convenient in modeling information flows of quantitative indicators to model the indicators mentioned above as indicators with a cubic structure to carry out various economic calculations, track their relationships in information systems and ensure the consistency of values of various indicators, etc. In order to achieve this, it is necessary to develop an appropriate mathematical apparatus for working with cubic structures.

Let us introduce the necessary mathematical concepts for modeling of economic quantitative indicators.

Definition 1. An entity is any abstracted named object that has a defined structure. An instance of an entity is a particular object that has a structure assigned by an entity and has specific values of attributes for this structure.

The term structure here means a set of required attributes of an entity. As you can see from the definition, a mandatory attribute of any entity is its name, that is, any entity has at least a name. There are other examples of attributes for entities such as an entity code, an entity version, an entity's unit of measure, other entities that are elements of a given entity, etc. For each entity these optional attributes are defined separately. For instance, you can define "the period" entity that has the following attributes: period type, period start date, period end date. An instance of the "the period" entity can be a period named "2020" and attributed values: type of the period = "year", start date of the period = "01/01/2020", end date of the period = "12/31/2020".

2. Space of factors

Definition 2. A factor is an entity that contains an ordered finite set of other entities, called factor values. The value of a factor is an entity that has a serial number.

An example of a factor is the "gender" indicator with a set of values "male sex", "female sex", or an indicator "type of traction" with a set of values "electric traction", "diesel traction", "steam traction". Thus, a factor is a named object that contains an ordered set of values of factors. The factors will be denoted by lowercase letters of the Latin alphabet, sometimes indicating in parentheses the number of factor values: $f, g, h(4)$, etc., and the factor values - by the same letters with subscripts, where the index shows the ordinal number of the factor value (we will assume that the values of the factors are numbered with natural numbers starting from one, although in practice this is not always the case).

Note 1. An important characteristic of a factor is that all values of a factor are different, that is, it cannot have two identical values.

Definition 3. If the sets of values for two factors coincide (without regard to the order), then they are found to be equal.

This definition implicitly contains the concept of equality of factor values, which is determined by the semantics of the subject area. Sometimes the names of the values of factors may differ, but they are considered equal to each other based on the characteristics of the subject area under consideration. For example, in the railway industry the factors "traction type" are considered with a set of values {"electric traction", "diesel traction"} and "electrification of a track section" with a set of values {"electrified section", "non-electrified section"}. These factors are considered to be equal, since electric trains always run-on electrified sections, and heat trains run on non-electrified sections. But if two factors are equal, then looking at Note 1, they always have the same number of values. The equality of the values of various factors are considered a priori given from the semantics of the subject area.

The values of factors are usually qualitative properties of objects of the subject area, therefore, another feature of the semantics of the subject area, which must be taken into account when working with factors, is the compatibility of the values of factors. For example, in the railway industry there is a factor "type of cargo" with values {"coal", "oil cargo", "timber cargo", "mineral fertilizers", "cereal", "metals", "household items"} and the factor "type wagon" with the values {"flat", "high sided", "covered wagon", "tank", "refrigerator", "other wagon"}. The values of both factors can characterize different objects of the subject area, but there is not a single subject area object that would be characterized simultaneously by two values, such as {"coal", "tank"}, {"oil cargo", "flat"} or {"household items", "tank"}, because coal cannot be transported in tanks, oil cargo on flat-wagons and household items in tanks. Such pairs of values of two factors are called inconsistent. The pair of values as {"oil cargo", "tank"} are joint, since such goods are transported only in tanks. Thus, we know a priori the joint and inconsistent values of the factors for an arbitrary pair of factors.

Definition 4. n values of n factors $\{f_1, g_2, \dots, h_n\}$ are given. A given set of factor values is inconsistent if there is at least one pair of incompatible values of the factors included in this set. Otherwise, the specified set of factor values is called joint.

Definition 5. Two factors f and g are called independent if all possible pairs of their values are compatible. Otherwise, the factors are called dependent.

Note 2. The independence of n factors is determined in a similar way.

Definition 6. $f = \{f_1, f_2, \ldots, f_n\}, g = \{g_1, g_2, \ldots, g_m\}$ are two factors with the indicated values of the factors. The sum of the factors f and g, is the factor $h = f + g$, the set of values of which is the union of the sets of values of the factors f and g, and the order of the values is determined by the order of the values of the factors in the arguments, that is, the factor h has values $n + m$ and $h_1 = f_1, \ldots, h_n = f_n, h_{n+1} = g_1, \ldots, h_{n+m} = g_m$.

Note 3. From definition 6 and Note 1 it follows that $f = f + f$, for any factor f, that is, the operation of adding factors is an idempotent operation.

Note 4. The addition of n factors is determined similarly. For instance,$f + g + h$ as $(f + g) + h$.

Lemma 1. (Properties of addition) The operation of adding factors has the following properties:

- $f + g = g + f$ for any factors as f and g (commutativity of addition).
- $(f + g) + h = f + (g + h)$ for any factors as f, g и h (associativity of addition).

The proof follows from the definition of addition.

Definition 7. Let $f^i = \{f_1^i, f_2^i, \ldots, f_{n_i}^i\}, i = 1,2,\ldots,k,$ k factors with the indicated values of the factors. The product of factors $f^i, i = 1,2,\ldots,k$ is called $h = f^1 \cdot f^2 \cdot\ldots\cdot f^k = \prod_{i=1}^k f^i$, a set of values of which are instances of the entity "factor value", which are formed from sets of values of arguments $\{(f_{i_1}^1, f_{i_2}^2, \ldots, f_{i_k}^k), i_1 = 1,2,\ldots,n_1, i_2 = 1,2,\ldots,n_2,\ldots, i_k = 1,2,\ldots,n_k\}$, excluding inconsistent sets of values. The order for values of product is determined by the order of the arguments and the order of the values of the arguments.

Example 1. $f = \{f_1, f_2, f_3\}$, $g = \{g_1, g_2\}$. Then $f \cdot g = \{(f_1, g_1), (f_1, g_2), (f_2, g_1), (f_2, g_2), (f_3, g_1), (f_3, g_2)\}$, and $g \cdot f = \{(f_1, g_1), (f_2, g_1), (f_3, g_1), (f_1, g_2), (f_2, g_2), (f_3, g_2)\}$.

Lemma 2. (Properties of multiplication) The operation of multiplying factors has the following properties:

- $f \cdot g = g \cdot f$ for any factors as f and g (commutative property of multiplication).
- $(f \cdot g) \cdot h = f \cdot (g \cdot h)$ for any factors as f, g and h (associativity of multiplication).
- $(f + g) \cdot h = f \cdot h + g \cdot h$ for any factors as f, g and h (distributivity over addition).

Proof of the Lemma 2. The commutative property is derived from the definition. Let us prove the associativity of multiplication. The values of the factor $(f \cdot g) \cdot h$ are triples (f_i, g_j, h_k), where the indices run through the numbers of the values of the corresponding factors. The same triplets constitute the set of values of the $f \cdot (g \cdot h)$, consequently, by definition, these factors are equal. Let us prove the third property. The set of values of factor $(f + g) \cdot h$ consists of pairs $(z_i, h_j), i = 1,2,\ldots,m + n, j = 1,2,\ldots,l$, where m, n, l – are the numbers of values for factors f, g, h respectively, z_i ranges over the sets of values of factors f and g. The set of values of the factor $f \cdot g + g \cdot h$ consists of the union of the sets $(f_i, h_j), i = 1,2,\ldots,m, j = 1,2,\ldots,l$ and $(g_i, h_j), i = 1,2,\ldots,n, j = 1,2,\ldots,l$. It is easy to understand that these sets coincide, consequently $(f + g) \cdot h = f \cdot h + g \cdot h$. Lemma 2 is proven.

Lemma 3. The number of factor values $\prod_{i=1}^k f^i$ is equal to $C = \prod_{i=1}^k n_i$, where n_i – number of values of i -factor, if factors $f^i, i = 1,2,\ldots,k$ are independent.

The proof is obvious.

Definition 8. $f^i = \{f_1^i, f_2^i, \ldots, f_{n_i}^i\}, i = 1,2,\ldots,k$ - the set of k factors with the indicated values of the factors. Cubic product (multiplication) of factors $f^i, i = 1,2,\ldots,k$ is the following factor

$$h = \sum_{i=1}^k \sum_{l=(l_1,l_2,\ldots,l_i)} \prod_{j=1}^i f^{l_j}, \quad (1)$$

where the summation by the internal sum is carried out by all possible sub-vectors $(l_1, l_2, \ldots, l_i)$ vector $(1,2,\ldots,k)$, where $l_1 < l_2 <\cdots< l_i$.

The cubic product of k factors is denoted as $f^1 * f^2 *\ldots* f^k$. When $k = 2$, it follows from Definition 8, that the cubic product of two vectors f and g is:

$$f * g = f + g + f \cdot g. \quad (2)$$

When $k = 3$ the cubic product of three factors as f, g, h, can be written as

$$f * g * h = f + g + h + f \cdot g + f \cdot h + g \cdot h + f \cdot g \cdot h \quad (3)$$

Example 2. Let $f = \{f_1, f_2, f_3\}, g = \{g_1, g_2\}, h = \{h_1, h_2, h_3\}$. Then

$$f * g * h = \left\{\begin{matrix} f_1, f_2, f_3, g_1, g_2, h_1, h_2, h_3, (f_1, g_1), (f_1, g_2), (f_2, g_1), (f_2, g_2), (f_3, g_1), (f_3, g_2), (f_1, h_1), \\ (f_1, h_2), (f_1, h_3), (f_2, h_1), (f_2, h_2), (f_2. h_3), (f_3, h_1), (f_3, h_2), (f_3, h_3), (g_1, h_1), (g_1, h_2), \\ (g_1, h_3), (g_2, h_1), (g_2, h_2), (g_2, h_3), (f_1, g_1, h_1), (f_1, g_1, h_2), (f_1, g_1, h_3), (f_1, g_2, h_1), \\ (f_1, g_2, h_2), (f_1, g_2, h_3), (f_2, g_1, h_1), (f_2, g_1, h_2), (f_2, g_1, h_3), (f_2, g_2, h_1), (f_2, g_2, h_2), \\ (f_2, g_2, h_3), (f_3, g_1, h_1), (f_3, g_1, h_2), (f_3, g_1, h_3), (f_3, g_2, h_1), (f_3, g_2, h_2), (f_3, g_2, h_3) \end{matrix}\right\}.$$

The term cubic product is justified by the fact that it has a cubic structure. Namely, if we construct a k -dimensional discrete space and set the values of the i -factor on each axis, then the values of the cubic product can be represented as the cells of a discrete k -dimensional cube. Indeed, in the formula (1) members under the outer sum for $i = 1$ represent the edges of a cube (one-dimensional faces of a cube), with $i = 2$ two– dimensional faces of a cube, etc., when $i = k$ – inside of the cube (k -dimensional faces of a cube). Thus, the set of values of a factor obtained as the cubic product of other factors can be represented as a list of values, or as cells of a cube of which dimension coincides with the number of factors involved in the cubic product.

The cubic product of factors is of great use in the construction of quantitative economic indicators, the so-called "meters".

Example 3. Consider the example of the cubic product of factors from the railway industry. There are the following factors: f = "type of message" = {"internal", "import", "export", "transit"} and g = "type of cargo shipment" = {"wagon", "group", "route", "small", "container"}. The cubic product of these factors can be represented as the following two-dimensional cube (Table 1):

Table 1. Result of the cubic product of two factors.

	Internal message (f_1)	Import message (f_2)	Export message (f_3)	Transit message (f_4)
Full-wagon shipment (g_1)	Full-wagon shipment in an internal message (f_1, g_1)	Full-wagon shipment in an import message (f_2, g_1)	Full-wagon shipment in an export message (f_3, g_1)	Full-wagon shipment in a transit message (f_4, g_1)
Group shipment (g_2)	Full-wagon shipment in an internal message (f_1, g_2)	Group shipment in an import message (f_2, g_2)	Group shipment in an export message (f_3, g_2)	Group shipment in a transit message (f_4, g_2)
Routed shipment (g_3)	Routed shipment in an internal message (f_1, g_3)	Routed shipment in an import message (f_2, g_3)	Routed shipment in an export message (f_3, g_3)	Routed shipment in a transit message (f_4, g_3)
Small consignment (g_4)	Small consignment in an internal message (f_1, g_4)	Small consignment in an import message (f_2, g_4)	Small consignment in an export message (f_3, g_4)	Small consignment in a transit message (f_4, g_4)
Container consignment (g_5)	Container consignment in an internal message (f_1, g_5)	Container consignment in an import message (f_2, g_5)	Container consignment in an export message (f_3, g_5)	Container consignment in a transit message (f_4, g_5)

On the Table 1, the values of the "type of message" factor are set aside by columns, and the values of the "type of cargo shipment" factor are set aside by rows, and the values of the $f * g$ factor are shown in the table cells.

Lemma 4. (Properties of the cubic product) The operation of the cubic product of factors has the following properties:

- $f * g = g * f$ for any factors as f and g (commutative property of cubic multiplication).
- $(f * g) * h = f * (g * h)$ for any factors as f, g and h (associativity of cubic multiplication).
- $(f + g) * h = f * h + g * h$ for any factors as f, g and h (distributivity of multiplication relative to addition).
- $(f + g) * h + f \cdot g * h = f * g * h$ for any factors as f, g and h (decomposability of cubic multiplication).

Proof of the Lemma 4. $f * g = f + g + f \cdot g = g + f + g \cdot f = g * f$. Here we have used the commutativity of multiplication and addition. We will prove the second property. Using the relation (2) we have $(f * g) * h = (f + g + f * g) + h + (f + g + f * g) * h = f + g + f * g + h + f * h + g * h + (f * g) * h = f + g + h + g * h + f(g + h + g * h) = f * (g * h)$. Let us prove the third property $(f + g) * h = f + g + h + (f + g) * h = f + g + h + f * h + g * h = f + h + f * h + g + h + g * h = f * h + g * h$. The last property is proved analogously. Lemma 4 is proved.

Note 5. The last property from the lemma shows that when two two-dimensional cubes are added together, a three-dimensional cube can be obtained.

Lemma 5. The factor $f = g^1 * g^2 * \ldots * g^l$ is considered to be the cubic product of factors as $g^1, g^2, \ldots, g^l$. Let us fix the first values of the k factors and consider the set of all the values of the factor f, that have the values of the first k factors $g^i, i = 1,2,\ldots,k$, fixed. The resulting set of values forms a factor that can be represented as a cubic product $g^{k+1} * g^{k+2} * \ldots * g^l$. The factor obtained in this way is called the slice of factor f and is denoted by $f\left[g^1_{c_1}, g^2_{c_2}, \ldots, g^k_{c_k}\right]$ or$g^1 * g^2 * \ldots * g^l\left[g^1_{c_1}, g^2_{c_2}, \ldots, g^k_{c_k}\right]$, where $g^1_{c_1}, g^2_{c_2}, \ldots, g^k_{c_k}$are fixed values of first k factors.

Proof of the Lemma 5. The set of all values for factor f, that have fixed first values of k factors, can be represented, according to formula (1), as following $\sum_{i=k+1}^{l} \sum_{c=(c_{k+1},c_{k+2},\ldots,c_i)} \cup \left(g^1_{c_1}, g^2_{c_2}, \ldots, g^k_{c_k}, g^{k+1}_{c_{k+1}}, \ldots, g^l_{c_l}\right) = \sum_{i=k+1}^{l} \sum_{c=(c_{k+1},c_{k+2},\ldots,c_i)} \prod_{j=k+1}^{l} \tilde{g}^{c_j} = \tilde{g}^{k+1} * \tilde{g}^{k+2} * \ldots * \tilde{g}^l$, where $\tilde{g}^{(k+i)}$ is a factor with values $\{g^1_{(c_1)}, g^2_{(c_2)}, \ldots, g^k_{(c_k)}, g^{(k+i)}_{(j_{(k+i)})}\}$, where $g^{k+i}_{j_{k+i}}$ runs through all the values of $g^{(k+i)}$ factor. Lemma 5 is proved.

Note 6. To obtain a slice of the cubic product, it is not necessary to fix the values of the first k factors, but you can fix the values of random k factors. To bring the lemma to the conditions, it is enough to reorder the factors so that the factors with fixed values are the first.

Slice of the cubic product plays an important role when working with meters [1].

3. Space of measurements

Definition 9. Measurements describe the quantitative properties of objects in the subject area. The values of the meter are measured in the units of the measurement.

As we noted at the beginning of the article, any quantitative economic indicators have a multidimensional structure, and this multidimensional structure is determined by the dependence of the measurement on the factor. Let I be the measurement, and $f = \{f_1, f_2, \ldots, f_n\}$ be the factor with the specified values on which this measurement depends. Then the values of this factor give rise to the measurement's subsidiary measurements with the same unit of measure, which we will call indicators. For example, consider the above example of a measurement I = “population” with a unit of measure “Person”, and a factor f = “Gender” = {“male”, “female”}. Then the I measurement will have the indicators I_1= “number of men”, I_2= “number of women”. The dependence of the measurement I on the factor f will be denoted by $I[f]$, and the corresponding indicators - $I_i = I(f_i)$, where i is the ordinal number of the value of the factor f_i. The value of the measurement for an object will be denoted by $Z(I)$, and the value of the measurement's indicator by $Z(I_i)$ or $Z\big(I(f_i)\big)$.

If the measurement depends on several factors $f^i = \{f^i_1, f^i_2, \ldots, f^i_{n_i}\}, i = 1,2,\ldots,k$, then let us define that dependence as $I[f^1 * f^2 * \ldots * f^k]$. The designation $I[f^1 * f^2 * \ldots * f^k]$ indicates that the measurement I has a list of subsidiary measurements generated by the values of the factor $f^1 * f^2 * \ldots * f^k$, but factor $f^1 * f^2 * \ldots * f^k$, as we mentioned above has a cubic structure, therefore, the indicators of the measurement can also be displayed in the form of a cube, each cell of which corresponds to a certain indicator. To emphasize that we consider the exponents of a measurement as the cells of some cube sometimes the measurement $I[f^1 * f^2 * \ldots * f^k]$ will be designated as $I[f^1, f^2, \ldots, f^k]$, the number of arguments in square brackets shows the number of dimensions of the cube. In this case, the designation $I[f^1, f^2, \ldots, f^k]$ means that we consider the measurement as a k-dimensional cube. Correspondingly, if we write this measurement as $I[f^1 * f^2 * \ldots * f^{k-1}, f^k]$, it means that, this measurement is considered as two-dimensional, the first dimension of which is formed by the values of the factor $f^1 * f^2 * \ldots * f^{k-1}$, and second by values of the factor f^k. The values of the indicators will be denoted by $Z\left(I\left(f_{i_1}^1, f_{i_2}^2, \ldots, f_{i_k}^k\right)\right)$, where $f_{i_1}^1, f_{i_2}^2, \ldots, f_{i_t}^t, t \le k$ fixed values of the relevant factors. If $t = k$, then the indicator is located inside the cube if $t < k$, then the indicator is located on the cube face or edge of the cube. As an example, consider the measurement "Cargo accepted for shipment" with the unit of measure "Ton", which depends on the factors "Type of message" and "Type of cargo shipment" from Example 3. Then the indicators of this measurement can be represented as the following two-dimensional cube (Table 2):

Table 2. The measurement "Cargo accepted for shipment" and its indicators.

Cargo accepted for shipment (I)	Cargo accepted for shipment in an internal message $I(f_1)$	Cargo accepted for shipment in an import message $I(f_2)$	Cargo accepted for shipment in an export message $I(f_3)$	Cargo accepted for shipment in a transit message $I(f_4)$
Cargo accepted for shipment by full-wagon $I(g_1)$	Cargo accepted for shipment in an internal message by full-wagon $I(f_1, g_1)$	Cargo accepted for shipment in an import message by full-wagon $I(f_2, g_1)$	Cargo accepted for shipment in an export message by full-wagon $I(f_3, g_1)$	Cargo accepted for shipment in a transit message by full-wagon $I(f_4, g_1)$
Cargo accepted for shipment by groups $I(g_2)$	Cargo accepted for shipment in an internal message by groups $I(f_1, g_2)$	Cargo accepted for shipment in an import message by groups $I(f_2, g_2)$	Cargo accepted for shipment in an export message by groups $I(f_3, g_2)$	Cargo accepted for shipment in a transit message by groups $I(f_4, g_2)$
Cargo accepted for shipment by routeways $I(g_3)$	Cargo accepted for shipment in an internal message by routeways $I(f_1, g_3)$	Cargo accepted for shipment in an import message by routeways $I(f_2, g_3)$	Cargo accepted for shipment in an export message by routeways $I(f_3, g_3)$	Cargo accepted for shipment in a transit message by routeways $I(f_4, g_3)$
Cargo accepted for shipment by small consignment $I(g_4)$	Cargo accepted for shipment in an internal message by small consignment $I(f_1, g_4)$	Cargo accepted for shipment in an import message by small consignment $I(f_2, g_4)$	Cargo accepted for shipment in an export message by small consignment $I(f_3, g_4)$	Cargo accepted for shipment in a transit message by small consignment (f_4, g_4)
Cargo accepted for shipment by container consignment $I(g_5)$	Cargo accepted for shipment in an internal message by container consignment $I(f_1, g_5)$	Cargo accepted for shipment in an import message by container consignment $I(f_2, g_5)$	Cargo accepted for shipment in an export message by container consignment $I(f_3, g_5)$	Cargo accepted for shipment in a transit message by container consignment $I(f_4, g_5)$

On the Table 2 the values of the factor “type of message” are displayed on columns, and the values of the factor “type of cargo dispatch” are displayed on rows. In the first cell of the first row there is a measurement, and in the other cells there are indicators of the measurement.

Note 7. The number of indicators for the measurement $I[f^1 * f^2 * \dots * f^k]$ equals to $\prod_{j=1}^{k}(n_j + 1) - 1$, where n_j is the number of values for j -factor, if given factors are independent.

Definition 10. $I[f^1 * f^2 * \dots * f^k]$ and $J[f^1 * f^2 * \dots * f^k]$ – are two measurements that depend on the same factors and with the same unit of measure. The sum (difference) of these measurements is $K[f^1 * f^2 * \dots * f^k]$ with the same unit of measure, with the value of measurement as $Z(K) = Z(I) \pm Z(J)$ and with values of indicators as

$$Z\left(K\left(f_{i_1}^1, f_{i_2}^2, \dots, f_{i_k}^k\right)\right) = Z\left(I\left(f_{i_1}^1, f_{i_2}^2, \dots, f_{i_k}^k\right)\right) \pm Z\left(J\left(f_{i_1}^1, f_{i_2}^2, \dots, f_{i_k}^k\right)\right), \tag{4}$$

where i_j indexes go from the ordinal numbers of values of the factor f^j.

Definition 11. It’s said that the measurement $I[f]$ does not depend on the factor f, if $Z(I(f_i)) = Z(I)$ for any $i = 1,2, \dots, n$, where n – the number of values for factor f.

This definition states that if the values of the indicators of the measurement do not change when the value of a certain factor changes, then the dependence of the measurement on this factor can be neglected. On the other hand, if the measurement does not depend on a certain factor, then we can assume that this measurement depends on this factor, but the values of the measurement indicators do not change when moving from one value of this factor to another value. That is, if the measurement I depends on the factors $f^1, f^2, \dots, f^k$, but does not depend on the factor f^{k+1}, then we can assume that the measurement I depends on all the factors $f^1, f^2, \dots, f^{k+1}$, and the corresponding values of the measurement indicators for the values of the factor f^{k+1} are determined by the formula

$$Z(I(f^1 * f^2 * \ldots * f^k, f_i^{(k+1)})) = Z(I(f^1 * f^2 * \ldots * f^k)), \tag{5}$$

where f_i^{k+1} go from the set of all values for factor f^{k+1}. This fact allows us to determine the sum (difference) of two measurements, that are depending on various factors.

Definition 12. The sum (difference) of the two measurements $I[f^1 * f^2 * \dots * f^k]$ and $J[g^1 * g^2 * \ldots * g^l]$ is given as $I[h^1 * h^2 * \dots * h^m] \pm J[h^1 * h^2 * \dots * h^m]$, where the set $\{h^1, h^2, \ldots, h^m\}$ is the union of sets $\{f^1, f^2, \ldots, f^k\}$ and $\{g^1, g^2, \ldots, g^l\}$.

Thus, $I[f] \pm J[g] = K[f, g]$, that is, the sum of two measurements, each of which depends on one factor, is a measurement that depends on two factors.

Note 7. The sum of n measurements is determined similarly. For example, $I + J + K$ is defined as $(I + J) + K$.

Lemma 6. (properties of sum) The operation of adding (difference) measurements has the following properties:

- $I \pm J = J \pm I$ for any measurements as I and J (commutative property of addition).
- $(I \pm J) \pm K = I \pm (J \pm K)$ for any measurements as I, J and K (associativity of addition).

The proof is simple.

Definition 13. $I[f^1 * f^2 * \dots * f^k]$ and $J[g^1 * g^2 * \dots * g^l]$ are two measurements with units of measures such as e_I and e_J respectively. The product of them is the measurement $K[h^1 * h^2 * \dots * h^m] = I[f^1 * f^2 * \dots * f^k] \cdot J[g^1 * g^2 * \dots * g^l]$ with the unit of measure $e_I \cdot e_J$, and the value of measurement $Z(K) = Z(I) \cdot Z(J)$ and with values of indicators as

$$Z\left(K\left(h_{i_1}^1, h_{i_2}^2, \dots, h_{i_s}^s\right)\right) = Z\left(I\left(h_{i_1}^1, h_{i_2}^2, \dots, h_{i_s}^s\right)\right) \cdot Z\left(J\left(h_{i_1}^1, h_{i_2}^2, \dots, h_{i_s}^s\right)\right), \tag{6}$$

where i_j indexes go from the number of orders of the factor h^j, s changes from 1 to m, the set $\{h^1, h^2, \dots, h^m\}$ is the unions of sets $\{f^1, f^2, \ldots, f^k\}$ and $\{g^1, g^2, \dots, g^l\}$.

Note 8. The product of n measurements is determined similarly. For example, $I \cdot J \cdot K$ is found to be $(I \cdot J) \cdot K$.

Lemma 7. (product properties) The operation of multiplying measurements has the following properties:

- $I \cdot J = J \cdot I$ for any measurements as I and J (commutative property of multiplication).
- $(I \cdot J) \cdot K = I \cdot (J \cdot K)$ for any measurements as I, J and K (associativity of multiplication).
- $(I \pm J) \cdot K = I \cdot K \pm J \cdot K$ for any measurements as I, J and K (distributivity over addition).

The proof comes from similar properties of numbers and the cubic product.

Definition 14. $I[f^1 * f^2 * \ldots * f^k]$ and $J[g^1 * g^2 * \ldots * g^l]$ are two measurements with units of measures such as e_I and e_J respectively and the set $\{h^1, h^2, \ldots, h^m\}$ is the union of sets $\{f^1, f^2, \ldots, f^k\}$ and $\{g^1, g^2, \ldots, g^l\}$. For any cell with the coordinates $h^1_{(i_1)}, h^2_{(i_2)}, \ldots, h^s_{(i_s)}$ or $Z\left(J\left(h^1_{i_1}, h^2_{i_2}, \ldots, h^s_{i_s}\right)\right) \neq 0$, or $Z\left(I\left(h^1_{i_1}, h^2_{i_2}, \ldots, h^s_{i_s}\right)\right) = 0$. Then the ratio of the measurements I and J is the measurement $K[h^1 * h^2 * \ldots * h^m] = \frac{I[f^1 * f^2 * \ldots * f^k]}{J[g^1 * g^2 * \ldots * g^l]}$ with the unit of measure $\frac{e_I}{e_J}$, and the value of measurement $Z(K) = \frac{Z(I)}{Z(J)}$ and with values of indicators such as $Z\left(K\left(h^1_{i_1}, h^2_{i_2}, \ldots, h^s_{i_s}\right)\right) = \begin{Bmatrix} \frac{Z\left(I\left(h^1_{i_1}, h^2_{i_2}, \ldots, h^s_{i_s}\right)\right)}{Z\left(J\left(h^1_{i_1}, h^2_{i_2}, \ldots, h^s_{i_s}\right)\right)}, & if & Z\left(J\left(h^1_{i_1}, h^2_{i_2}, \ldots, h^s_{i_s}\right)\right) \neq 0 \\ 0, & & Z\left(J\left(h^1_{i_1}, h^2_{i_2}, \ldots, h^s_{i_s}\right)\right) = 0 \end{Bmatrix}$,

where i_j indexes go from the ordinal numbers of values of the factor h^j, s changes from 1 to m.

Definition 15. $I[f^1 * f^2 * \ldots * f^l]$ is the measurement that depends on factors as $f^1, f^2, \ldots, f^l$ and $f^1 * f^2 * \ldots * f^l\left[f^1_{i_1}, f^2_{i_2}, \ldots, f^k_{i_k}\right]$ is the slice of the cubic product of factors. Then measurement $I\left[f^1 * f^2 * \ldots * f^l\left[f^1_{i_1}, f^2_{i_2}, \ldots, f^k_{i_k}\right]\right]$ is the slice for measurement $I[f^1, f^2, \ldots, f^l]$.

Lemma 8. The slice of measurement $I\left[f^1 * f^2 * \ldots * f^l\left[f^1_{i_1}, f^2_{i_2}, \ldots, f^k_{i_k}\right]\right] = I(f^1_{(i_1)}, f^2_{(i_2)}, \ldots, f^k_{(i_k)})$, that is, any slice of the measurement coincides with some indicator of this measurement.

The proof is obvious.

Note 9. As follows from Lemma 8, the slice of any measurement is also a measurement that depends on those factors which values are not fixed for this indicator. If we consider the measurement as a multidimensional cube, then the slice of the measurement corresponds to the operation of the slice of a multidimensional cube.

An important property of measurements is the type of a measurement. By type, the measurements are divided into integral measurements and differential measurements.

Definition 16. The measurement $I[f^1 * f^2 * \ldots * f^l]$ is called integral, if

$$Z(I) = \sum_{j=1}^{n_i} Z\left(I\left(f^i_j\right)\right), \tag{7}$$

for any $i = 1,2, \ldots, l$, and any indicator (or a slice) of the measurement I has the (7) property. Otherwise, the measurement $I[f^1 * f^2 * \ldots * f^l]$ is called to be differential.

The integrality of the measurement means that the value of the measurement itself and any indicator of this measurement is equal to the sum of the direct "children" of this indicator by any factor on which this indicator depends. Integral measurements are usually quantitative economic indicators, and differential ones are qualitative economic indicators.

The measurement I from the Table 2 is the integral measurement, because $Z(I) = Z\big(I(f_1)\big) + Z\big(I(f_2)\big) + Z\big(I(f_4)\big) + Z\big(I(f_4)\big) = Z\big(I(g_1)\big) + Z\big(I(g_2)\big) + Z\big(I(g_4)\big) + Z\big(I(g_4)\big) + Z\big(I(g_5)\big)$ and each subtrahend of these equalities has an analogous property.

Lemma 9. The sum and difference of two integral measurements is an integral measurement.

Proof of the Lemma 9. The lemma is sufficient to prove for the measurement itself. $I[f^1 * f^2 * \ldots * f^k]$ and $J[f^1 * f^2 * \ldots * f^k]$ are measurements that depend on the same factors. Then by the formula (7), $\sum_{(j=1)}^{(n_i)} Z((I+J)(f_j^i)) = \sum_{(j=1)}^{(n_i)} Z(I(f_j^i)) + \sum_{(j=1)}^{(n_i)} Z(J(f_j^i)) = Z(I) + Z(J) = Z(I+J)$. Here we used equation (4) twice [2]. Lemma 9 is proved.

4. Conclusion

The proposed model of multidimensional economic indicators is a convenient tool for solving various economic problems. The appropriate database structure for storing such indicators allows us to monitor the consistency of the values of the indicators during the data collection process. The proposed multidimensional structure of economic indicators reflects their internal nature.

Using the given mathematical apparatus we developed a factor-balance method for calculating the cost of services and an information system for calculating the cost of cargo transportation by rail for the national railway company of the Republic of Kazakhstan. An important advantage of the developed methodology compared with similar solutions is the ability to take into account the impact on the cost of various factors, such as “type of traction”, “type of message”, “type of cargo”, “type of wagon”, “category of train”, “category of shipment”, “distance of transportation”, etc. The information system for calculating the cost of services was also developed for JSC “National Information Technologies”, which is a national company of the Republic of Kazakhstan that provides information technology services.

In our opinion, the use of multidimensional indicators provides additional advantages to the researcher in solving any economic and social problems.

References

[1] Gabbassov M B 2014 TOFI technology capabilities for data processing and visualization *Collection of Conf. Materials “Application of Information and Communication Technologies-AICT2014” (Nur-Sultan*) pp 276-77

[2] Gabbassov M B, Quanov T D and Abilkayeva Zh N 2017 *Theoretical foundations of TOFI technology and data exchange* (Nur-Sultan) p 149