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To cite this article: Bakyt Shalabayeva *et al* 2019 *J. Phys.: Conf. Ser.* **1391** 012121

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Mathematical model of the motion of suspended particles in a turbulent flow and their influence on the motion of an unmanned aerial vehicle

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Abstract. A mathematical model of turbulent motion of a non-homogeneous flow is constructed. Based on the pulsation energy balance method, a closed system of equations is obtained for calculating the average velocity and turbidity of a non-uniform flow. The calculation of the pulsation characteristics of the flow with a transverse shift is carried out and the analysis of the effect of the impurity on the pulsation structure of the turbulent flow and its effect on the motion of an unmanned aircraft is carried out. Since the safety assessment and service lifetime of an unmanned aerial vehicle cannot exclude the influence of disturbances in the atmosphere when calculating characteristics.

1. Introduction

The problem of moving suspended particles in a turbulent flow is of considerable interest, both from theoretical and practical points of view, due to various technical applications: hydraulic excavation, movement of sediments in rivers, movement of dust and other gases. As an example, flight tests are the most reliable method for studying the flight of UAVs but in practice various methods for analytic studies of the equations of controlled movement UAVs are widely used. The more mathematically described the studied phenomena and processes, the more difficult the system of equations of the controlled motion of the aircraft and the more difficult the study of this system. Therefore, always follow the path of the greatest possible simplification of the



equations allowed by the task. One of the main factors affecting on the flight dynamics of an aircraft is that it is exerted by various disturbances, which, as a rule, are turbulent pattern [1].

Particles of loose, solid material carried by water and air currents are collectively called sediment. They are produced by hydrodynamic forces generated during the flow of liquid or air environment and their movement in suspension is due to turbulent mixing, usually accompanied by real flow. The specific gravity of the sediments, except for some special cases, varies within a small range from 2.2 to 2.6. Their dimensions are assumed to be small enough so that the probability of maintaining them in suspension is large. This definition of small particles is conditional because it is closely related to the value of the longitudinal component of the velocity and intensity of turbulence so that higher values allow larger particles to be suspended. Thus, the main characteristic of suspensions is not so much the geometric size but the speed of its fall through the fluid. A particle moving in flow has a net vertical movement due to the difference between u_2 the vertical component of velocity and a the speed of the gravitational falling where the rate of gravitational settling, a , is called "hydraulic size."

Basic to the study of the motion of suspended particles is the question of the transport capacity of the flow, i.e. to calculate, for the hydraulic characteristics of a given flow and for sediments of a specified size, whether the fluid is able to lift and move the sediment in a suspended state for a predetermined distance.

Fundamentals of the theory of motion of suspended sediments were laid Velikanov [2, 3]. He writes : "*The whole process of suspended of sediments is closely connected with the turbulence of the flow, so the level of knowledge about the movement of sediments at a given height is a problem of turbulence.*" So it is natural to use the fluctuating energy balance method to derive the equations of turbulent motion of suspended particles. The theory, was developed further by Kolmogorov [4] and Barenblatt [5]. The basic assumption of this theory is the smallness of the size of the suspended particles (compared to the characteristic scale of the turbulence), suggest that they form a kind of continuously distributed impurity in the main fluid.

Theory motion of suspended particles in a turbulent flow needs to be developed. As since a semiempirical theory introduced ideas which may be used in gravitational theory.

In this paper, we present a closed system of equations for the average velocity and turbidity for inhomogeneous flow on the basis of the single-point second-order moments. We use these equations to calculate the pulsation characteristics of the shear flow including the analysis of the influence of impurities on the fluctuating structure of the turbulent flow.

2. General Equations

We will consider the equations motion of non-homogeneous fluid for laminar flow conditions by following [5]:

$$\begin{aligned} \frac{\partial U_i}{\partial \tau} + U_k \frac{\partial U_i}{\partial x_k} &= -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_k \partial x_k} - (1 + \sigma S) g_i, \\ \frac{\partial S}{\partial \tau} + U_k \frac{\partial S}{\partial x_k} &= a \frac{\partial S}{\partial x_2}, \\ \frac{\partial U_k}{\partial x_k} &= -a\sigma \frac{\partial S}{\partial x_2}. \end{aligned} \quad (2.1)$$

Where (x_1, x_2, x_3) are Cartesian coordinates with the x_2 axis directed vertically upwards, τ is the time, U_i are the components of the fluid velocity, S is the relative volume of suspended solids or turbidity (turbidity - volume concentration), P is the total hydraulic pressure of uniform fluid, ρ is the fluid density, ν is the kinematic viscosity, σ is the mass of suspended particles, g_i are the components of the acceleration of gravity, and a is the rate of uniform sedimentation of a single particle in an infinite space filled with fluid and is assumed constant.

We consider the case of small relative volumes of suspended particles and small mass within the stream, i.e.

$$S \ll 1, \quad \sigma S \ll 1, \quad \sigma = \frac{\rho_2 - \rho_1}{\rho}. \quad (2.2)$$

Where ρ_1 is the density of the liquid and ρ_2 is the suspended matter, respectively.

The suspended particles in an, on average, horizontal flow, are explained by the presence of the vertical component of the pulsation of velocity flow. Horizontal laminar flow can not carry particles if the particles are so large that molecular diffusion can be neglected. It follows that of special interest for the problem is turbulent motion in an inhomogeneous fluid. In this case, we use the averaged equations of motion given by replacing the quantities of interest by their expectation values. The resulting equations in the ergodic hypothesis are approximately valid for motion characteristics averaged over periods of time which are small compared with the characteristic time for the mean flow and large compared with the characteristic time of fluctuations [6, 7].

To move to the averaged equations, we give U_i , P , S as the sum of the averaged and pulsation components [6, 7]:

$$U_i = \bar{U}_i + u_i; \quad S = \bar{S} + s; \quad P = \bar{P} + p. \quad (2.3)$$

We substitute these values in equations (2.1). In that case we obtain the equation for the mean and fluctuating components of velocity (sign averaging below discarded, where we have defined $\partial_k \equiv \frac{\partial}{\partial x_k}$):

$$\partial_\tau \bar{U}_i + \bar{U}_k \partial_k \bar{U}_i + \partial_k \langle u_i u_k \rangle = -\frac{1}{\rho} \partial_i \bar{P} + \nu \nabla^2 \bar{U}_i - (1 + \sigma \bar{S}) g_i - a\sigma \langle u_i \partial_2 s \rangle, \quad (2.4)$$

$$\partial_\tau u_i + \bar{U}_k \partial_k u_i + u_k \partial_k \bar{U}_i + \partial_k [u_i u_k - \langle u_i u_k \rangle] = -\frac{1}{\rho} \partial_i p + \nu \nabla^2 u_i - a\sigma [u_i \partial_2 s - \langle u_i \partial_2 s \rangle] - \sigma s g_i, \quad (2.5)$$

the equation for the mean and the fluctuating components of suspended matter:

$$\partial_\tau \bar{S} + \bar{U}_k \partial_k \bar{S} + \partial_k \langle u_k s \rangle = a \partial_2 \bar{S} - \frac{a\sigma}{2} \partial_2 \langle s^2 \rangle, \quad (2.6)$$

$$\partial_\tau s + \bar{U}_k \partial_k s + u_k \partial_k \bar{S} + \partial_k [u_k s - \langle u_k s \rangle] = a \partial_2 s - \frac{a\sigma}{2} \partial_2 [s^2 - \langle s^2 \rangle], \quad (2.7)$$

the equation of continuity for the mean and fluctuating velocity:

$$\begin{aligned} \partial_k U_k &= -a\sigma \partial_2 \bar{S}, \\ \partial_k u_k &= -a\sigma \partial_2 s. \end{aligned} \quad (2.8)$$

As in [6, 7, 8], in order to close the system of equations (2.4)-(2.8) we will use the equations for the single-point second-order moments of velocity fields and turbidity fields. The equation for the fluctuating component of velocity u_j will be written similar to the equation (2.5). Then multiply both sides of equation (2.5) on u_j , and both sides of the equation for u_j - on u_i . After we sum up these two equations and as a result, after averaging and transformation, we obtain the equation for the velocity correlation:

$$\begin{aligned} \partial_\tau \langle u_i u_j \rangle + U_k \partial_k \langle u_i u_j \rangle + \langle u_k u_j \rangle \partial_k U_i + \langle u_i u_k \rangle \partial_k U_j = \partial_k \left[\nu \partial_k \langle u_i u_j \rangle - \langle u_i u_j u_k \rangle - \left\langle (\delta_{jk} u_i + \delta_{ik} u_j) \frac{p}{\rho} \right\rangle \right] \\ + \left\langle \frac{p}{\rho} (\partial_i u_j + \partial_j u_i) \right\rangle - 2\nu \langle \partial_k u_i \partial_k u_j \rangle - \sigma g (\delta_{i2} \langle s u_j \rangle + \delta_{j2} \langle s u_i \rangle) - 2a\sigma \langle u_i u_j \partial_2 s \rangle, \end{aligned} \quad (2.9)$$

Multiplying equation (2.5) on s and multiplying equation for s (2.7) on u_i . After we sum up these two equations and as a result, after averaging, we obtain the equation for the correlation of suspended matter:

$$\begin{aligned} \partial_\tau \langle s u_i \rangle + U_k \partial_k \langle s u_i \rangle + \langle u_k s \rangle \partial_k U_i + \langle u_k u_i \rangle \partial_k S \\ = \partial_k \left[\nu \partial_k \langle u_i s \rangle - \langle u_i u_k s \rangle - \left\langle \frac{p}{\rho} s \right\rangle \delta_{ik} \right] + \left\langle \frac{p}{\rho} \partial_i s \right\rangle - 2\nu \langle \partial_k s \partial_k u_i \rangle - \sigma g \delta_{i2} \langle s^2 \rangle + a \langle u_i \partial_2 s \rangle, \end{aligned} \quad (2.10)$$

the equation for the mean-square fluctuation of turbidity obtained from (2.7) will have the form:

$$\partial_\tau \left\langle \frac{s^2}{2} \right\rangle + U_k \partial_k \left\langle \frac{s^2}{2} \right\rangle + \langle u_k s \rangle \partial_k S = \partial_k \left[\nu \partial_k \frac{s^2}{2} - \left\langle u_k \frac{s^2}{2} \right\rangle \right] + a \partial_2 \left\langle \frac{s^2}{2} \right\rangle. \quad (2.11)$$

3. The main hypotheses

We now use the main hypotheses of Kolmogorov-Rotta [8, 9, 10, 11] and their analogues for the relative volume of suspended particles in developed turbulent flow:

$$\begin{aligned} \left\langle \frac{p}{\rho} (\partial_j u_i + \partial_i u_j) \right\rangle &= -k \frac{\sqrt{E}}{l} \left(\langle u_i u_j \rangle - \frac{2}{3} \delta_{ij} E \right), \\ \left\langle \frac{p}{\rho} \partial_i s \right\rangle &= -k_s \frac{\sqrt{E}}{l} \langle u_i s \rangle, \\ 2\nu \langle \partial_k u_i \partial_k u_j \rangle &= \frac{2}{3} \delta_{ij} c \frac{E^{\frac{3}{2}}}{l}, \\ 2\nu \langle \partial_k u_i \partial_k s \rangle &= 0 \\ E &= \frac{1}{2} (\langle u_1^2 \rangle + \langle u_2^2 \rangle + \langle u_3^2 \rangle). \end{aligned} \quad (3.1)$$

Here k , k_s , and c are empirical constants [7, 9, 10] (the evaluation of which is given below), E is the kinetic energy of the pulsation motion of fluid, l has the dimensions of length (displacement path).

We will assume that the structure close to isotropic turbulence [7] so that in the

$$\langle u_i \partial_2 s \rangle = 0. \quad (3.2)$$

Since $\langle s^2 \rangle < S$ and $\sigma < 1$ we also have,

$$\partial_2 \left\langle \frac{s^2}{2} \right\rangle \ll \partial_2 S. \quad (3.3)$$

In order to find a semi-empirical algebraic representation of the quantity $\partial_2 \langle s^2 \rangle$ we note that the characteristics of the turbulent flow depend on the averaged fluctuation of energy E , the value of length l , and its gradient of velocity U_2 . Also, it is clear that the gradient of $\langle s^2 \rangle$ is proportional to the size of $\langle s^2 \rangle$ itself and has a negative sign since the turbidity, as well as its mean square fluctuation, decrease with increasing height. The simplest assumption that satisfies the above considerations is

$$l^2 \partial_2 u \partial_2 \langle s^2 \rangle = -c_s \sqrt{E} \langle s^2 \rangle, \quad (3.4)$$

where c_s is the only empirical constant related to the presence of impurities in the stream.

Further, we will assume that the inhomogeneous flow is a fully developed turbulent flow, also we will neglect the third-order moments and the transfer of any substance pressure pulsations. This allows us to drop the diffusion terms from equations (2.9)-(2.11) as evidenced by experimental data, which show that substantial diffusion ripple currents only occur near the axis [7]. Furthermore, we will assume that the inhomogeneous fluid is pure shear, ie. $u = U(y)$, $w = v = 0$, $S = S(y)$. Finally, we obtain the system for the average velocity and volume concentration:

$$\begin{aligned} \partial_k \langle -u_i u_k \rangle + (1 + \sigma S) g_i &= 0, \\ \partial_k \langle -s u_k \rangle - a \partial_2 S &= 0. \end{aligned} \quad (3.5)$$

The system of equations of one-point second-order moments (2.9)-(2.11) with (3.1) can be written as follows:

$$\begin{aligned} \partial_\tau \langle u_i u_j \rangle + \langle u_k u_j \rangle \partial_k U_i + \langle u_i u_k \rangle \partial_k U_j + k \frac{\sqrt{E}}{l} \left(\langle u_i u_j \rangle - \frac{2}{3} \delta_{ij} E \right) + \frac{1}{3} c \delta_{ij} \frac{E^{\frac{3}{2}}}{l} + \sigma g (\delta_{2j} \langle s u_i \rangle + \delta_{i2} \langle s u_j \rangle) &= 0, \\ \partial_\tau \langle s u_i \rangle + \langle u_k s \rangle \partial_k U_i + \langle u_k u_i \rangle \partial_k S + k_s \frac{\sqrt{E}}{l} \langle u_i s \rangle + \sigma g \delta_{i2} \langle s^2 \rangle &= 0, \\ \partial_\tau \left\langle \frac{s^2}{2} \right\rangle + \langle u_k s \rangle \partial_k S - a \partial_2 \left\langle \frac{s^2}{2} \right\rangle &= 0. \end{aligned} \quad (3.6)$$

Thus, (3.5) and (3.6) are the basic equations of a mathematical model for the turbulent motion of an inhomogeneous fluid under sufficiently general assumptions about the nature of the movement.

4. Calculation of pulsation characteristics developed turbulent flow of an inhomogeneous fluid.

We will consider turbulent flow of an inhomogeneous fluid, as noted earlier, we assume pure shear. To calculate the pulsation characteristics we separate equation (3.6) into components,

where we define $(x_1, x_2, x_3) \rightarrow (x, y, z)$ and $(u_1, u_2, u_3) \rightarrow (u, v, w)$:

$$\begin{aligned}
\langle uv \rangle \frac{\partial U}{\partial y} + \frac{k\sqrt{E}}{2} \frac{1}{l} \left(\langle u^2 \rangle - \frac{2}{3}E \right) + \frac{1}{3}c \frac{E^{\frac{3}{2}}}{l} &= 0, \\
\frac{k\sqrt{E}}{2} \frac{1}{l} \left(\langle v^2 \rangle - \frac{2}{3}E \right) + \frac{1}{3}c \frac{E^{\frac{3}{2}}}{l} + \sigma g \langle sv \rangle &= 0, \\
\frac{k\sqrt{E}}{2} \frac{1}{l} \left(\langle w^2 \rangle - \frac{2}{3}E \right) + \frac{1}{3}c \frac{E^{\frac{3}{2}}}{l} &= 0, \\
\langle v^2 \rangle \frac{\partial U}{\partial y} + \frac{k\sqrt{E}}{2} \frac{1}{l} \langle uv \rangle + \sigma g \langle su \rangle &= 0, \\
\langle vw \rangle \frac{\partial U}{\partial y} + \frac{k\sqrt{E}}{2} \frac{1}{l} \langle uw \rangle &= 0, \\
\frac{k\sqrt{E}}{2} \frac{1}{l} \langle vw \rangle &= 0, \\
\langle vs \rangle \frac{\partial U}{\partial y} + \langle vu \rangle \frac{\partial S}{\partial y} + k_s \frac{\sqrt{E}}{l} \langle us \rangle &= 0, \\
\langle v^2 \rangle \frac{\partial S}{\partial y} + k_s \frac{\sqrt{E}}{l} \langle vs \rangle + \sigma g \langle s^2 \rangle &= 0, \\
\langle vs \rangle \frac{\partial S}{\partial y} + ac_s \frac{\sqrt{E}}{l^2} \frac{\partial U}{\partial y} \langle s^2 \rangle &= 0.
\end{aligned} \tag{4.1}$$

Empirical constants are not associated with the inhomogeneity of the fluid, and their determination is a matter of pure fluid in the flow. Therefore, we will assume that the conditional suspension has the same density as the fluid, i.e. $\sigma = 0$. In that case, the system of equations (4.1) allows us to express all the relative second-order moments of the velocity field and the volume concentration through the gradient of velocity, the gradient of turbidity, scale of length and empirical constants:

$$\begin{aligned}
\langle u^2 \rangle_0 &= \frac{2}{3} \left(1 + 2\frac{c}{k} \right) \frac{1}{c^{\frac{2}{3}}} \left(l \frac{\partial U}{\partial y} \right)^2, \\
\langle v^2 \rangle_0 &= \langle w^2 \rangle_0 = \frac{2}{3} \left(1 - \frac{c}{k} \right) \frac{1}{c^{\frac{2}{3}}} \left(l \frac{\partial U}{\partial y} \right)^2, \\
\langle -uv \rangle_0 &= \left(l \frac{\partial U}{\partial y} \right)^2, \\
E_0 &= \frac{1}{c^{\frac{2}{3}}} \left(l \frac{\partial U}{\partial y} \right)^2, \\
\langle -sv \rangle_0 &= \frac{k}{k_s} \left(l^2 \frac{\partial U}{\partial y} \frac{\partial S}{\partial y} \right), \\
\langle su \rangle_0 &= \frac{c^{\frac{1}{3}}}{k_s} \left(1 + \frac{k}{k_s} \right) \left(l^2 \frac{\partial U}{\partial y} \frac{\partial S}{\partial y} \right), \\
\frac{k}{k_s} l^2 \frac{\partial U}{\partial y} \left(\frac{\partial S}{\partial y} \right)^2 &= ac_s \frac{1}{c^{\frac{1}{3}}} \frac{\langle s^2 \rangle}{l}, \\
\langle s^2 \rangle_0 &= \frac{k}{k_s} \frac{1}{c_s a} l^2 \frac{\partial U}{\partial y} \left(\frac{\partial S}{\partial y} \right)^2.
\end{aligned} \tag{4.2}$$

Note that the solutions (4.2) contain all the pulsation characteristics of the turbulent flow for homogeneous fluid and denoted by the subscript "0". The numerical coefficient of the expressions $\langle -uv \rangle_0$ in (4.2) is equal to unity, since the shear scale of hypotheses (4.1) is defined up to a constant factor. Such ideas used in [7,9,10], and these ideas allow us to express both empirical constants k and c in one $\frac{k}{c} = 7$, i.e. we will reduce the number of experimental constants.

In order to determine the effect of particulate matter in the stream on the pulsation characteristics, the solutions of (4.1) easily will be found in the form of two factors $M = M_0\psi$ (M_0 coincides with the solution (4.2), ψ reflects the influence of impurities). Thus, ψ should be determined by a parameter of interaction, where $\psi(0) = 1$.

We will now confine ourselves to $\langle uv \rangle$ and $\langle sv \rangle$ the expression needed to calculate the velocity and turbidity of the medium.

$$\langle -uv \rangle = \left(l \frac{\partial U}{\partial y} \right)^2 \psi_4, \quad \langle -sv \rangle = \frac{k}{k_s} \left(l^2 \frac{\partial U}{\partial y} \frac{\partial S}{\partial y} \right) \psi_5, \quad (4.3)$$

Where ψ_i, ψ_4, ψ_5 - functions of pulsating motion, which take into account the effect of (impurity) suspension. Then the solutions (4.2) can be written as follows:

$$\begin{aligned} E &= E_0\psi, & \langle u^2 \rangle &= \langle u^2 \rangle_0 \psi_1, & \langle v^2 \rangle &= \langle v^2 \rangle_0 \psi_2, & \langle w^2 \rangle &= \langle w^2 \rangle_0 \psi_3, \\ \langle uv \rangle &= \langle uv \rangle_0 \psi_4, & \langle vs \rangle &= \langle vs \rangle_0 \psi_5, & \langle su \rangle &= \langle su \rangle_0 \psi_6, & \langle s^2 \rangle &= \langle s^2 \rangle_0 \psi_7. \end{aligned} \quad (4.4)$$

Substituting these solutions into equation (4.1) we obtain a system of algebraic equations for the function ψ_i :

$$\begin{aligned} -\psi_4 + \frac{\alpha}{3} \sqrt{\psi} \left[\left(1 + \frac{2}{\alpha} \right) \psi_1 - \psi \right] + \frac{1}{3} \psi^{\frac{3}{2}} &= 0, \\ \frac{\alpha}{3} \sqrt{\psi} \left[\left(1 - \frac{1}{\alpha} \right) \psi_2 - \psi \right] + \frac{1}{3} \psi^{\frac{3}{2}} - \frac{Ri}{\sigma_T} \psi_5 &= 0, \\ \frac{\alpha}{3} \sqrt{\psi} \left[\left(1 - \frac{1}{\alpha} \right) \psi_3 - \psi \right] + \frac{1}{3} \psi^{\frac{3}{2}} &= 0, \\ \psi_2 - \sqrt{\psi} \psi_4 + \frac{c^{\frac{3}{2}}}{kk_s} \left(1 - \frac{1}{\sigma_T} \right) Ri \psi_6 &= 0, \\ \frac{1}{\sigma_T} \psi_5 + \psi_4 &= \left(1 + \frac{1}{\sigma} \right) \sqrt{\psi} \psi_6, \\ \psi_2 - \sqrt{\psi} \psi_5 + \frac{1}{\omega} Ri \psi_7 &= 0, \\ \psi_5 &= \sqrt{\psi} \psi_7 \\ Ri &= -\sigma g \frac{S_y}{U_y^2}, \quad \omega = k_s c_s \frac{a}{c^{\frac{2}{3}} l U_y}, \quad \alpha = \frac{k}{c}. \end{aligned} \quad (4.5)$$

Ri is the dimensionless parameter is analogous to the Richardson number and represents a measure of the influence of impurities. If the flow is homogeneous or contains a uniformly distributed mixture, then the number $Ri = 0$ and the function ψ must be equal to one $\psi = 1$. The parameter ω is the ratio of the speed of gravitational sedimentation to the dynamic speed, α is the empirical constant [7, 9, 10, 11]. We will express the function ψ_i in terms of ψ , the parameters of interaction, and the empirical constants:

$$\begin{aligned}
\psi_1 &= \psi + \frac{Ri}{\sigma(\alpha+2)} \frac{\psi}{\psi + \frac{Ri}{\omega} + \frac{3Ri}{\sigma(\alpha-1)}}, \\
\psi_2 &= \frac{\psi^2 + \frac{Ri\psi}{\omega}}{\psi + \frac{Ri}{\omega} + \frac{3Ri}{\sigma(\alpha-1)}}, \\
\psi_3 &= \psi \\
\psi_4 &= \psi^{\frac{3}{2}} \frac{\frac{Ri}{\sigma} \psi^{\frac{3}{2}}}{\psi + \frac{Ri}{\omega} + \frac{3Ri}{\sigma(\alpha-1)}}, \\
\psi_5 &= \frac{\psi^{\frac{3}{2}}}{\psi + \frac{Ri}{\omega} + \frac{3Ri}{\sigma(\alpha-1)}}, \\
\psi_6 &= \frac{\psi}{(\sigma+1)} \frac{1+Ri}{\psi + \frac{Ri}{\omega} + \frac{3Ri}{\sigma(\alpha-1)}}, \\
\psi_7 &= \frac{\psi^2}{\psi + \frac{Ri}{\omega} + \frac{3Ri}{\sigma(\alpha-1)}}.
\end{aligned} \tag{4.6}$$

ψ is then determined from the equation:

$$\psi^2 + \psi \left[\frac{Ri}{\sigma} \left(\frac{\sigma}{\omega} + \frac{9}{2(\alpha-1)} + 1 \right) - 1 \right] + \frac{3Ri^2}{2\sigma^2(\alpha-1)} \left[\frac{1}{Ri} + 1 + \frac{\sigma}{\omega} + \frac{3}{\alpha-1} \right] - \frac{Ri}{\omega} = 0. \tag{4.7}$$

which can be written in the form:

$$\begin{aligned}
\psi &= \frac{1}{2} \left[-B + \sqrt{B^2 - 4C} \right], \\
B &= \frac{Ri}{\sigma} \left(\frac{\sigma}{\omega} + \frac{9}{2(\alpha-1)} + 1 \right) - 1, \\
C &= \frac{3Ri^2}{2\sigma^2(\alpha-1)} \left[\frac{1}{Ri} + 1 + \frac{\sigma}{\omega} + \frac{3}{\alpha-1} \right] - \frac{Ri}{\omega}.
\end{aligned} \tag{4.8}$$

Thus, we have found all of the functions ψ_i for the single-point second-order moments. In Figures 1, 2, and 3 we present the results of the numerical calculation for the basic functions of pulsatile motion. As with the experimental data [7, 12] we see from Figures 1, 2, 3 that changes occur due to the action of Ri , the Richardson number and the parameter ω .

From the expression (4.8) it is clear that ψ will vanish for a particular combination of parameters. This means that at a certain limit or critical value Ri^* of the Richardson number, the parameter ω^* is suppressed in pulsatile motion. These values are determined from the expression for $\psi = 0$:

$$Ri^* = \frac{\alpha-1}{3} \frac{2\sigma^2(\alpha-1) - \omega}{\sigma(\alpha-1) + \omega^*(\alpha+2)}. \tag{4.9}$$

If $\omega \ll 1$, the settling velocity of particles is small, the critical Ri^* becomes

$$Ri^* = \frac{2}{3} (\alpha-1) \approx 3. \tag{4.10}$$

For $\omega \rightarrow \infty$, i.e. very large suspended particles, there is almost instantaneous suppression of the turbulent motion:

$$Ri^* \rightarrow 0. \tag{4.11}$$

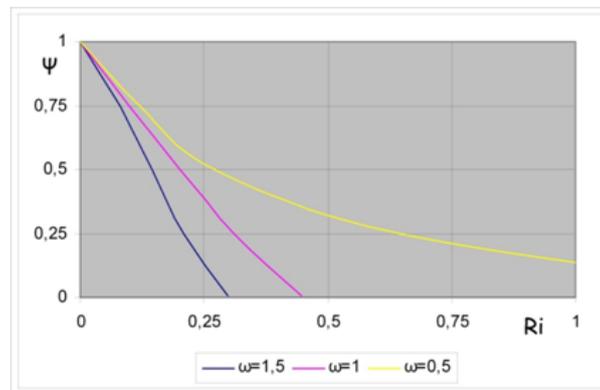


Figure 1. Distribution for the function ψ in turbulent fluctuations at different values of the Richardson number Ri and ω the parameter of suspended particles with $\omega = 0.5, 1, 1.5$ respectively.

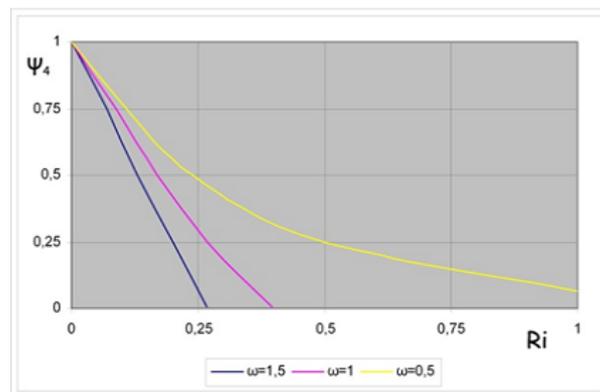


Figure 2. Distribution of the pulsating movement function ψ_4 at different Ri and the parameter $\omega = 0.5, 1, 1.5$ respectively.

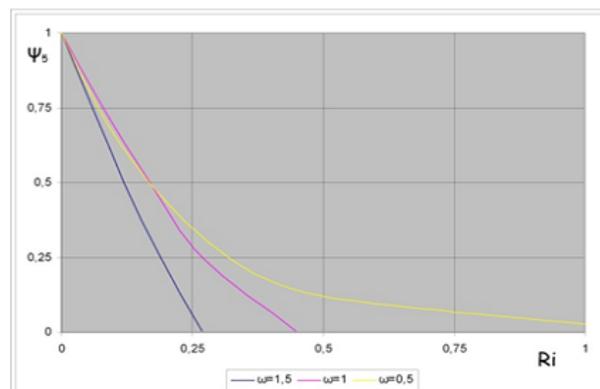


Figure 3. Distribution of the function ψ_5 according to Ri and $\omega = 0.5, 1, 1.5$ respectively.

5. Conclusion

The paper presents a mathematical model of turbulent motion of a non-uniform flow, which satisfies general assumptions about the nature of motion, because the method of turbulent pulsations is based on measurements of gas concentrations and three-dimensional wind speed, and flows are calculated based on these measurements. The equations of the closed system for the average velocity and turbidity are obtained on the basis of the equations for the second-order single-point moments. In addition, a numerical calculation of the shear flow pulsation characteristics was performed and an analysis of the effect of impurities on the pulsation structure of the turbulent flow and its effect on the motion of an UAVs was given. When various factors are affected by the UAVs (condition of indeterminacy, nonlinear aerodynamic parameters, the pattern of elastic vibrations of the structure, effects and interference in the atmosphere) arising from a relatively low flight speed, the UAVs are very sensitive to disturbances: atmospheric turbulence, movements of suspended particles in turbulent flow, etc. In the atmosphere of atmospheric turbulence, due to the interaction between the UAV and the air flow during the flight, a number of flexural-torsional vibrations occur, which leads to the destruction of the UAV design [13, 14].

Acknowledgments

The authors are grateful to the supervisor Dzhaugashtin Kazbek Esenzhanovich for valuable recommendations in the set up and execution of this research.

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