PAPER • **OPEN ACCESS**

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To cite this article: H Y Barminova and R Eshchanova 2020 *J. Phys.: Conf. Ser.* **1690** 012049

Model analysis of relativistic electron beam dynamics in a rarefied plasma

H Y Barminova[1,2] and R Eshchanova[3,4]

[1] National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoye sh. 31, Moscow, 115409, Russian Federation
[2] Peoples Friendship University of Russia (RUDN University), Miklukho-Maklaya Str. 6, Moscow, 117198, Russian Federation
[3] Peoples Friendship University of Russia (RUDN University), Miklukho-Maklaya Str. 6, Moscow, 117198, Russian Federation (on leave)
[4] L.N.Gumilyov Eurasian National University, Satpayev Str. 2, Nur-Sultan, 010008, Republic of Kazakhstan

E-mail: eebarminova@mephi.ru

Abstract. A mathematical model for describing the nonlinear dynamics of a relativistic electron beam in a rarefied plasma is discussed. The model can be applied for the analysis of the behavior of an electron beam in accelerators, as well as in electromagnetic radiation generators based on relativistic electron beams. The equation for the beam envelope and the results of its numerical solution are presented and shortly discussed.

1. Introduction

The dynamics of relativistic electron beams (REB) has long been the subject of careful study due to the active use of relativistic electron beams in fundamental and applied research as well as in industry. Ongoing interest in REB study is now strongly stimulated around the world due to novel scientific projects which use relativistic electron beams as a key instrument in high energy physics experiments (circular and linear lepton colliders, photon colliders) and in generation of electromagnetic radiation (free-electron lasers (FEL), synchrotron light sources, novel microwave generators, THz-radiation sources)[1-5]. We should mention here the large modern projects like Future Circular Collider (FCCee) and CLIC at CERN, CEPC, International Linear Collider (ILC), KEKB, VEPP-4, DAΦNE, Elettra, MAX IV, ESRF, Soleil, SwissFEL, XFEL and many others. The interest in the study of relativistic electron beams is also stimulated by the development of new schemes and methods of particle acceleration, such as plasma wakefield accelerators (PWFA), laser wakefield acceleration (LWFA), various hybrid acceleration schemes. Here one should mention the global European projects Eupraxia [6] and ELI [7].

The dynamics of a relativistic electron beam moving in low-density plasma is of particular interest, since neutralization of the beam own space charge in a rarefied background plasma can lead to stabilization of the beam radius, but, on the other hand, the beam dynamics becomes nonlinear, and various beam-plasma instabilities can occur as well (see, for instance, [8,9]). Situations in which the beam moves in plasma environment are realized both in traditional accelerators (residual gas in beampipe, secondary emission, multiple-beam acceleration, etc.) and in novel accelerators based on the methods of collective acceleration. Some types of generators of electromagnetic radiation are plasma-assisted or plasma-based and operate on the principle of beam-plasma interaction. In general, the beam-plasma interaction changes the characteristics of the beam, so that the conditions of the experiment or the parameters of the device (installation) may differ from the expected ones.

The complexity of studying the behavior of a relativistic electron beam in a plasma is that the beam dynamics becomes essentially nonlinear. Nonlinear REB dynamics is usually studied by means of numerical modeling, in most cases using the PIC-method, but numerical experiments do not provide the complete qualitative picture of the processes affecting the beam dynamics. Analytical methods for studying the beam behavior allow us to estimate the scale of physical processes and the qualitative dependence of the beam characteristics on all the parameters of the problem (see [10-20]). In this paper the analytical model, which describe the nonlinear dynamics of a relativistic electron beam with current less than that of Alfven, is discussed. To simplify the analysis, we consider a stationary problem.

2. Envelope equation for the relativistic electron beam

Let us consider a relativistic electron beam moving in plasma environment. In approximation of a long axial-symmetric beam with uniform particle density for the case of quasineutral regime of the beam propagation we can write the equation for the beam particle transverse oscillations:

$$\frac{d\vec{q}}{dz} = -\omega^2(z)\vec{r}\ . \tag{1}$$

Here $\vec{q} = \frac{\vec{r}'}{\sqrt{1+\vec{r}'^2}}$, $\vec{r}' = \frac{d\vec{r}}{dz}$, $\vec{r}$ - transverse radius-vector of the beam particle, corresponding to the beam cross-section, z - longitudinal coordinate, $\omega^2(z) = \frac{i}{R^2}$, R - the beam radius, $i = \frac{J}{J_A}$, J - full current of the beam, J_A - the Alfven current, $J_A = \frac{mc^3}{e}\gamma_0\beta_0$, γ_0 and β_0 - relativistic factors of the beam, *m* and *e* - the mass and the charge of the electron respectively, *c* - the light velocity.

If $i << 1$, we obtain that $\vec{r}\,' << 1$ and the oscillations of the beam particle are linear with variable frequency.

The opposite case ($i \to 1$) corresponds to the strong nonlinearity of the particle oscillations.
One can write the beam current density j_z as follows:

$$j_z = -ec\kappa \int \frac{cp_z}{H} F dp \tag{2}$$

Here p_z is longitudinal component of the particle momentum, *p* - full momentum of the particle, *H* - full energy of the particle, *F* - kinetic distribution function, κ - the constant of normalization.
Then let us consider the case of the same full energy H_0 for all the particles in the beam:

$$F = f\delta(H - H_0),$$

where δ is delta-function. Assuming that for all the particle the next relation is fulfilled: $p_z > 0$, i.e. there are no particles moving in opposite direction inside the beam, and transverse velocities of the particles are fairly small, one can write the invariant for the equation (1):

$$I = \frac{(\ddot{\vec{q}}u - \vec{q}\ddot{u})^2}{\varepsilon^2} + \frac{\vec{q}^2}{u^2},$$

where ε - the constant corresponding to the beam transverse emittance, *u(z)* - additional function corresponding to the beam radius (as will be shown later). Note here that at first the invariant *I* was used in [18].

Then assuming that the distribution function *f* looks as

$$f = \delta(I - 1)$$

for the beam current one can obtain the next expression:

$$I = \frac{\pi e c^2 \kappa}{\left(\dot{u}^2 + \frac{u^2}{\varepsilon^2}\right)} \sigma \left(1 - \frac{r^2}{\dot{u}^2 + \frac{u^2}{\varepsilon^2}}\right) \quad (3)$$

From (3) it is easy to obtain the expression for the beam radius:

$$R^2 = \dot{u}^2 + \frac{\varepsilon^2}{u^2}$$

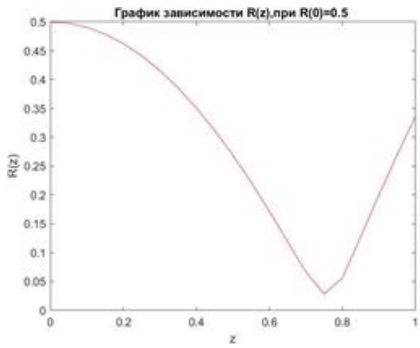
Finally, for the beam radius the following expression may be obtained:

$$\sqrt{1 + \frac{R'^2}{2} + \frac{\varepsilon^2}{2R^2}} \frac{d}{dz}\left(R' \sqrt{1 + \frac{R'^2}{2} + \frac{\varepsilon^2}{2R^2}}\right) + \frac{i}{R\sqrt{1 + \frac{R'^2}{2} + \frac{\varepsilon^2}{2R^2}}} = \frac{\varepsilon^2}{R^3}. \quad (4)$$

3. Numerical solution

Equation (4) is solved in MATLAB using 4th-order Runge-Kutta integration method, and a wide range of initial parameters is considered to study the behavior of the beam envelope before the Alfven current.

Some results of numerical solution of the equation (4) are shown at figures 1 and 2.



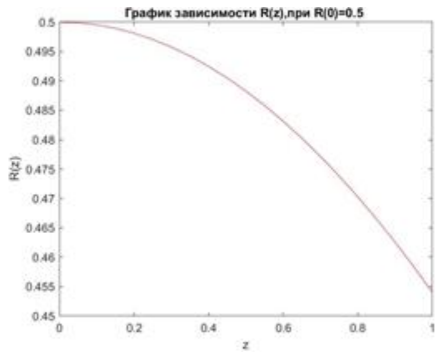


Figure1. Dependence of the REB radius on the longitudinal coordinate *z*. Initial conditions are: R(0) = 0.5, R'(0)=0. Value of the emittance ε=1, the beam current i=0.01.

Figure 2. Dependence of the REB radius on the longitudinal coordinate *z*. Initial conditions are: R(0) = 0.5, R'(0)=0. Value of the emittance ε=1, the beam current i=1.0.

Figures 1 and 2 show the dependence of the beam radius on the longitudinal coordinate *z* for the same initial values of the radius and its derivative. The values of the emittance in both cases are the same. The different behavior of the beam radius *R(z)* is explained by the difference in the beam current values. Figure 1 corresponds to the case of a beam current that isn't close to the Alfven current, and figure 2 corresponds to a current that is close to the Alfven current, and we see the effect of beam squeezing. This is a critical area where the application of equation (4) is limited.

4. Conclusion

Nonlinear dynamics of relativistic electron beam propagating in rarefied plasma is discussed. A self-consistent model based on the exact solution of the Vlasov equation allows us to analytically describe the dynamics of the beam depending on the beam initial parameters. For the case of the beam current below the Alfven current a nonlinear equation for the beam envelope may be obtained. In general case this equation must be solved by means of numerical integration. The results of numerical solution of the equation for the beam radius using MATLAB are presented which allow us to estimate the behavior of the beam envelope in a self-consistent field in cases of small and big difference between beam and Alfven current. The model discussed in this paper makes it possible to predict the beam squeezing in dependence on the beam current, beam initial radius and beam rms emittance. The results of the numerical solution show the limits of the model applicability.

Acknowledgements
The publication has been supported by the NRNU MEPhI 5/100 Program of Russian Academic Excellence Project.
The research has been supported by the RUDN University Strategic Academic Leadership Program.

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