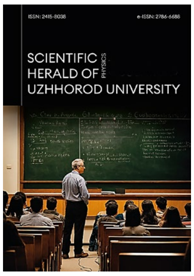
Scientific Herald of Uzhhorod University
Series "Physics"

Journal homepage: https://physics.uz.ua/en

Issue 55, 1975–1984

Received: 21.10.2023. Revised: 03.01.2024. Accepted: 18.03.2024



DOI: 10.54919/physics/55.2024.197vy5

On the essence and main components of the research approach in teaching mathematics at school

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Abstract

Relevance. The modern stage of social development is characterized by the rapid progress of science, which is expressed in deep study of mathematical methods for modelling natural phenomena and various processes, that is, a lot of attention is paid to methods of mathematical modelling.

Purpose. This paper studies the problem that without an appropriate research approach in teaching mathematics, i.e., without the organization of the educational process and the implementation of critical analysis and cognitive thinking, it is impossible to form a high level of critical thinking.

Methodology. The main method for conducting the study was the method of content analysis, and other general theoretical methods were also used.

Results. For a detailed study of critical analysis and cognitive thinking, some works of researchers concerning the thinking process are cited. At the same time, research is being carried out in the areas of critical analysis and cognitive thinking, which are confirmed by various examples. In conclusion, it must be said that only a thorough analysis can lead to a more complete study of the topic. The directions of critical analysis and cognitive thinking, as well as their interactions, will be shown on specific examples of educational school material. It is clear that critical analysis and cognitive thinking requires development and improvement.

Suggested Citation:
Mussaibekov R, Kappassova S, Atayev B, Rakhimzhanov B, Suleimenov K. On the essence and main components of the research approach in teaching mathematics at school. *Sci Herald Uzhhorod Univ Ser Phys.* 2024;(55):1975-1984. DOI: 10.54919/physics/55.2024.197vy5

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Conclusions. The practical significance of the study lies in the formation of specific characteristics of critical analysis and cognitive thinking within the research approach.

Keywords: critical analysis; counterexample; signs of equality of triangles; arithmetic and geometric progressions; cognitive thinking.

Introduction

The main goals of school education are to form skills and abilities, to gain in-depth knowledge, while the main attention should be paid to the independent activity of students, since one of the main approaches in the formation of knowledge is the active participation of students themselves. The achievement of these main goals is realized by the direct development of critical thinking, which is based on critical analysis and cognitive thinking of the issue under study and their further development through a research approach in teaching. The formation of a high level of critical thinking is impossible without the development of critical analysis and cognitive thinking, as well as without the ability to introduce research quest. On the other hand, without appropriate research in teaching mathematics in class and out-of-class time, it is impossible to develop critical analysis and cognitive thinking, and the formation of research skills.

Mastering the methods of mathematical modelling, as well as their further development, requires a high level of mathematical education, and school mathematics is the initial stage of fundamental mathematical education. In school mathematics, the formation of many mathematical methods begins, it is also called the mathematical apparatus. Therefore, not only understanding the work of mathematical methods, but also mastering them plays an important role for further studies in theoretical and applied mathematics.

Some scientists have previously studied both the formation of research skills in schoolchildren and the features of studying mathematics at school. For example, authors in the article studied empirically how solving problems in mathematics affects the formation of research competence in students [1]. In the course of the study, schoolchildren solved problems according to a methodology specially developed by the authors of the study. As a result, the goal was achieved, as schoolchildren, firstly, began to show great interest in solving problems, and secondly, improved their skills in problem-solving situations. A. B. Zhanys et al. also emphasize the importance of the development of thinking in schoolchildren in the process of learning mathematics, but they study another aspect of this problem [2]. The authors focus on the formation of aesthetic vision in schoolchildren by solving problems in mathematics. During the study, students solve the same problem in several different ways. According to the authors of the study, it develops students' creative skills, creative thinking, as well as an interest in mathematics R. Mussaibekov examined the concepts of “critical analysis” and “developmental analysis” [3]. The article provides various definitions of the indicated concepts, and also analyses various methods of teaching mathematics. The authors conclude that the solution of mathematical problems plays an important role in the development of the student's critical thinking.

K. Zhadyra et al. in the study emphasise the importance of properly teaching the elements of probability theory and statistics to Kazakhstani school students [4]. The article examines the peculiarities of teaching mathematics, discusses some textbooks and teaching methods. However, the research is limited to certain topics within the mathematics curriculum. Researchers in the field of mathematics education, J. Visnovska and J. L. Cortina, also examined various teaching methods of mathematics in schools in their study [5]. The authors provide practical recommendations for addressing issues in the designated teaching domain and highlight the importance of combining effective teaching methods with project-based learning. However, the use of a research approach in teaching mathematics at school among Kazakh scientists has not been studied enough. The paper aims to examine the research approach in teaching school mathematics and to analyze its components in order to obtain the necessary knowledge, skills and abilities for further study of methods of mathematical modeling.

Materials and Methods

As the material base of the study, examples from textbooks on mathematics for secondary school for 7-11 grades [6] and 8 grade [7] by authors from different decades were used. These textbooks were chosen because they are used in mathematics lessons in Kazakh schools. In order to study the text of these textbooks, content analysis was used. Content analysis is a research methodology used to systematically analyse and interpret the characteristics, patterns, and meanings of various forms of communication, such as text, images, audio, or video. It involves systematically examining the content of a communication, identifying and categorizing its elements, and deriving meaningful insights from the data. In this study, content analysis was used to select examples that were more relevant for the implementation of research goals from the textbooks in mathematics. As examples, such topics and concepts from the course of mathematics were used, such as definitions of “trapezoid”, section of “Progressions”, section of “Geometric Progression”, section of “Elements of Probability Theory”, concept of the “Limit of a function at a point”, etc. These concepts, definitions and topics were chosen because they are treated differently in different textbooks.

Using the methods of analysis and synthesis, the components of the concepts of “research approach”, “critical analysis” and “cognitive thinking” were studied, and then the author's definition of these concepts was formed. Also, with the help of these methods, the theoretical foundations of the problem under study were investigated, such as the role of the teacher in the understanding of mathematics by the student or the qualities that the teacher must possess in order to use the research approach in the classroom.

First, with the help of content analysis, examples from the mathematics course suitable for research were selected.

The selection criteria were the following: a concept or topic from a school mathematics course; a concept or topic has contradictions in its interpretation or understanding; a concept or topic can encourage the use of critical analysis and cognitive thinking in learning. Further, in order to correctly reveal the features of the formation and development of critical analysis and cognitive thinking in mathematics lessons, the main conditions for the implementation of a research approach in teaching were considered. After preparing the material and theoretical base, the analysis of the selected concepts and topics was carried out, the selected examples were considered in detail using their graphic and symbolic display. Using the method of analysis, the relationship between the study of mathematical concepts and the formation of skills of critical analysis and cognitive thinking in schoolchildren was revealed. After analysing all selected examples using the synthesis method, definitions of the concepts of "critical analysis" and "cognitive thinking" were formulated.

Results and Discussion

The process of organizing and carrying out research activities consists of teaching students the basic methods of analysing the problems under study, and it is also necessary to be guided by a deep understanding of the methods of mathematical modelling used. This process includes teaching students the main methods of reasoning, the choice of mathematical research methods, which are theoretical or empirical, that is, it is necessary to equip with an educational and cognitive apparatus. In the course of research activities, students will learn to experiment, compare and generalize facts; on the basis of the research, students will learn to think creatively, master the methods and styles of thinking, as well as master the methods of computer implementation in a complex, are the most important requirement for school graduates. When asked what is the motive of the study, one can answer that there must be a certain interest, a desire for creative and cognitive activity [8].

The use of a research-based approach in mathematics lessons is one way to develop critical analysis skills and cognitive thinking. Critical analysis is a process of evaluating information, arguments, or ideas in a systematic and objective manner. It involves examining the evidence and reasoning presented, identifying the strengths and flaws of the arguments, and assessing the overall validity and relevance of the information. Cognitive thinking is a complex and multifaceted process that underlies many aspects of human behaviour, including problem-solving, decision-making, creativity, and learning. It is influenced by both genetic and environmental factors, and can be improved through practice and experience [9].

Critical analysis and cognitive thinking are closely related, while in the course of their practical implementation, the analysis can be started from the critical, and then will smoothly move on to the developmental one. The level of formation of the implementation of critical analysis and cognitive thinking, as well as the skills of their practical implementation, accordingly affects the formation of the level of critical thinking itself. Another important problem is the preparedness of the teacher themselves, since the implementation of critical analysis and cognitive thinking sets high demands on the knowledge, skills and abilities of the mathematics teacher. Here are some requirements for the knowledge, skills and abilities of a mathematics teacher (they can be expanded or refined) for the correct organization and conduct of analyses:

– the prospective development of individual sections of the school mathematics course can be expressed in interdisciplinary communication, and can also be implemented in mathematical science;

– determination of the directions of possible generalization of the input parameters, as well as the organization of the corresponding study;

– development of a mathematical model for describing and studying economic, physical and other processes, the implementation of which is possible within the framework of school mathematics with slight complications of the type of mathematical model.

Consider an example of educational material in the study of which essential conditions for the use of the research approach [6; 7]. To substantiate the above areas of critical and developmental analyses, as well as to assess the degree of influence of their implementation on the level of critical thinking, consider some sections of the school mathematics course and carry out the appropriate implementation of critical and developmental analyses.

1. The definition of "trapezoid" can be considered as an example of the accuracy of the determination conditions. There are various definitions in the current textbooks:

a) in the textbook, the definition is given in the form: "A trapezoid is a quadrilateral in which only two opposite sides are parallel" [6]. This definition has redundant information – since an arbitrary quadrangle can only have opposite sides parallel, i.e., the term "opposing" is redundant. When formulating the definition, one could dispense with the concept of "opposing";

b) in the textbook, the definition is given as follows: "A trapezoid is a quadrilateral in which only two sides are parallel, and the other two are not parallel" [7].

There are these definitions. The main object is a quadrangle, and in which all pairs of sides are considered, then there are only 6 pairs of them, for example, the sides of a quadrilateral denote by {a, b, c, d}, then the following pairs of sides are considered: (a, b) (a, c), (a, d); (b, c) (b, d), (c, d). A. V. Pogorelov immediately separates pairs of sides by their location, namely, opposite sides, there will be two of the whole set of pairs [6]. V. A. Smirnov and E. A. Tuyakov do not consider such a division, they have taken a different approach. An example of a quadrangle is given below (Figure 1) [7].

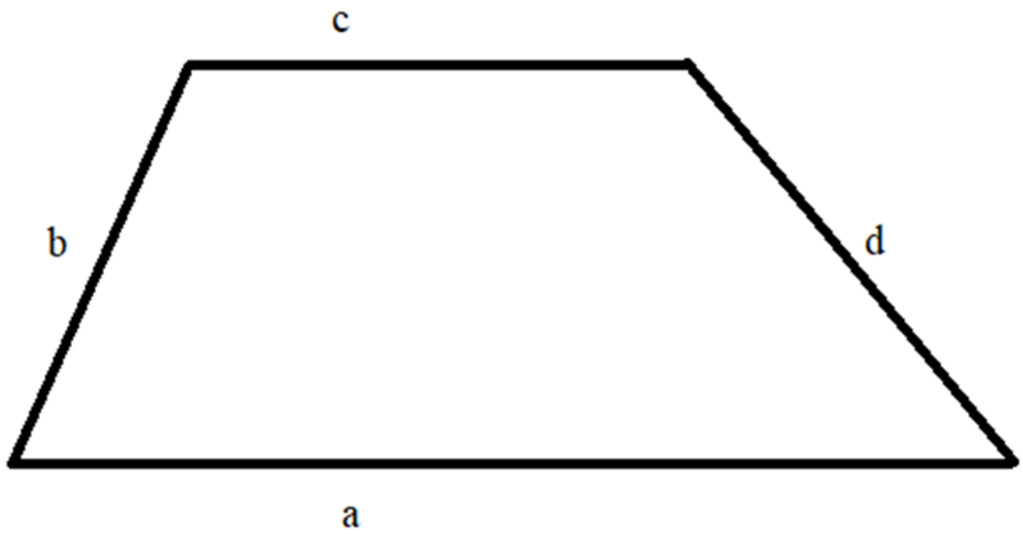

Figure 1. A quadrilateral
Source: [7].

In this figure of a quadrilateral, (a; c) can be taken as two parallel sides, then as "the other two", i.e., the second pair of sides can only be taken (b, d). The following should be a more precise definition: "A trapezoid is a quadrilateral in which only two sides are parallel". With this formulation, one can analyse all pairs of sides of a quadrilateral, namely, (a, b) (a, c), (a, d); (b, c) (b, d), (c, d). This definition will cover the definition given by A. V. Pogorelov [6], as well as V. A. Smirnov and E. A. Tuyakov [7].

2. The section of "Progressions" can be referred to the analysis of terminology. So, the arithmetic progression consists of two terms. The term "progress" in economics means the growth of a certain section of the economy by the same level, and in mathematics means a change by the number d. The term "arithmetic" is associated with the fulfilment of the property of the arithmetic mean, more precisely: $\forall\, n \in N$ and (1):

$$\forall\, k \in n \Longrightarrow a_n=(a_{n-k}+a_{n+k})/2. \quad (1)$$

3. Similarly for the section of "Geometric Progression". Another example related to terminology might be the dot product example of vectors. First, the term "scalar" is adopted from the fact that the product of vectors is a real number. Secondly, there is another product of two vectors, namely, the vector product of vectors, which is studied in university mathematics. In this example, the "dot product of vectors" carries additional information, one of them shows the nature of the result, and the second emphasizes the ambiguity of the product of vectors. On the other hand, the implementation of critical and developmental analyses can lead not only to a complete understanding of the educational material, but also to a deeper understanding of it, i.e., can determine the role of input parameters for the conclusion of the statement, to capture the effect of changing parameters to change the conclusion. In addition, it can allow for understanding of the future development of this topic in the framework of school mathematics and mathematical science.

The preparedness of a school graduate with an appropriate level of culture of thinking may be a sufficient level of preparedness to study the methods of mathematical modelling, as well as research methods of mathematical science. The application of critical and developmental analysis to some sections can lead to the emergence of a reverse reaction, namely, there may be a need for a deeper study of it for future development.

The paper considers such sections, as well as examples of the corresponding analyses. An example of such a section would be "Elements of Probability Theory". This educational material is introduced in school mathematics in order to familiarize oneself with a large "Probability Theory" theoretical section of mathematical science. Typically studied only simple cases of the space of elementary events, i.e. (2):

$$\Omega=\{\omega_1, \omega_2, \ldots, \omega_n\} \text{ and } p(\omega_1)=p(\omega_2)=\cdots=p(\omega_n)=1/n. \quad (2)$$

Here is one of the options for complicating the study material. Let the experiment consist of tossing a die. Then (3):

$$\Omega_1=\{\omega_1, \omega_2, \ldots, \omega_6\}. \quad (3)$$

In this case (4):

$$p(\omega_1)=p(\omega_2)=\cdots=p(\omega_6), p(\omega_n)=1/6. \quad (4)$$

Now let's take two dice, then (5):

$$\Omega_1=\{\omega_{11}, \omega_{12}, \ldots, \omega_{66}\}, \quad (5)$$

and (6):

$$p(\omega_{11})=p(\omega_{12})=\cdots=p(\omega_{66}), p(\omega_n)=1/36, \quad (6)$$

and it can be continued on. Further, in separate lessons, can be considered simple examples for the conditional probability, for the Bernoulli formula (a sequence of independent tests), etc. An instructive example about the relationship between probability and relative frequency (7):

$$P(A)=m/n \text{ and } W(A)=n_k/n. \quad (7)$$

There is the concept of the "Limit of a function at a point". The limit of a function at a point is determined by specifying the neighbourhood of the point (8):

$$U_\delta (x_0)= \{x \in R: |x- x_0| < \delta\}, \quad (8)$$

in this case, a "punctured" neighbourhood is considered (9):

$$\dot{U}_\delta (x_0)=\{x \in R: 0<|x- x_0| < \delta\}. \quad (9)$$

In the course of mathematical analysis, the limit of a function is determined through $\dot{U}_\delta (x_0)$ (punctured edge), then through the neighbourhood $U_\delta (x_0)$ the continuity is determined. Hence, the limit of the function f at the point $x_0$does not depend on the value of $f(x_0)$, even if $\exists\ f(x_0)$ or $\bar{\exists}\ f(x_0)$, but depends on a number of values, more precisely, on the behavior of the number of accepted f(x) values as x infinitely approaches x_0. The continuity of functions at the point x_0 means the fulfilment of an additional condition: at $x=x_0$, $f(x)=f(x_0)$ is fulfilled.

In some textbooks on algebra and the beginnings of analysis, in the case when x_0 point is not included in the domain of definition of functions, but is the limit point of the domain of definition, the simplest methods of finding are used, and the result of the true value of the limit is declared as a "limit value", such an approach is "harmful" for training, as it does not cover the definition of the limit. At the same time, the definition of the limit is replaced by a special case, namely, the definition of continuity at a point.

This approach is dictated by the need to bypass the exact definition of the limit of a function at a point, which is given in the course of mathematical analysis, and which is difficult for perception and comprehension in school mathematics. On the other hand, this approach loses the relationship between the absolute errors between the values: $|x-x_0| < \delta$ and $|f(x)-A_0| < \varepsilon$ or in case of continuity $|x-x_0| < \delta$ and $|f(x)- 〚f(x〛_0)| < \varepsilon$ for the purpose of filling such a gap between approaches, it is necessary to determine the purpose of studying the limit of a function at a point in the course of school mathematics. It is clear that such a goal is to introduce the concept of a derivative. Based on this goal, it can be studied the following ratio: $|x-x_0| < \delta$ and $|f(x)- 〚f(x〛_0)| < \varepsilon$, by means of well-chosen examples in which the parameters (ε, δ (ε))) will be related. Some complications, starting with sequence limits, can be explored in extracurricular activities or considered as a topic of research activity. This approach, for the ideal case, allows to preserve the structure of the definition of the limit and the relationship between the parameters (ε, δ (ε))).

The implementation of a critical analysis in clarifying the conditions of the statement can be the study of the equality of two triangles. Geometric definition implies the use of a motion transform. Two triangles ΔABC and $\Delta A_1 B_1 C_1$ are called equal if when performing certain types of motion transformation, i.e., in some sequence, these triangles will coincide. Transformations of motion are parallel translation, rotation through a certain angle, axial or central symmetry. It is clear that such transformations can be used more in theoretical reasoning than in practice, more precisely, analytically.

In addition to transforming motions on a plane, the analytical definition of the equality of two triangles is also used. The choice of a triangle is also natural due to the fact that any polygon on the plane can be represented by the union of a finite number of triangles, therefore, applying the appropriate tiles for two triangles, one can determine their equality. The analytical definition is based on the following equalities.

Table 1. The analytical definition of the equality of two triangles

Lengths of corresponding sides	Corresponding angles
$\left.\begin{array}{r} 1)\ a=a_1 \\ 2)\ b=b_1 \\ 3)\ c=c_1 \end{array}\right\}$	$\left.\begin{array}{r} 4)\ \alpha=\alpha_1 \\ 5)\ \beta=\beta_1 \\ 6)\ \gamma=\gamma_1 \end{array}\right\}$

The main task is to determine the fulfilment of the minimum number of equalities so that the fulfilment of the basic equalities can be proved. This topic also proves the fact that the number of six equalities performed can be reduced to three. In the first feature, two sides and the angle between them are taken, i.e., fulfilment of three equalities, for example, $a=a_1, b=b_1, \gamma=\gamma_1$. The research approach to studying this statement in its formulation is to check the accuracy of the chosen angle. Problem statement: if a different angle is chosen, will the conclusion of the statement be preserved?

In the course of analysing the accuracy of the choice of the angle between the sides, authors come to the statement that the choice of another angle (in the general case) negatively affects the conclusion of the statement. This research approach refers to critical analysis, and the construction of a counterexample leads to a triangle that is not equal to the original triangle, that is, the conclusion of the statement given will change.

A right-angled triangle gives the preservation of the conclusion of the statement. Indeed, in the case of a right-angled or isosceles triangles, the number of equalities decreases to two, because one equality is included in the definition of the very form of a triangle, so for a right-angled triangle one angle is known for sure, and for an isosceles triangle – two angles are equal and two sides are equal. This study also covers the tasks of constructing with a compass and a ruler. Such an analysis can be defined as developing, since the types of triangles are studied for which a change in conditions will not affect the conclusion, and the reason must be established. In the second sign of equality of triangles, one side and two adjacent angles are taken, in this case, analysing the derivation of two arbitrary angles convinced of the validity of the conclusion. This case is remarkable in that the conditions on the sides and angles can be arbitrary, i.e., for one side, any pair of angles can be taken, while the conclusion of the statement will not change, in the future such cases can be called the equivalence of these information. In the third sign, the equalities of the three corresponding sides are taken. The analysis of the choice of three corresponding angles influences the conclusion of the statement, which leads to a new concept, namely, to the similarity of triangles. It can be argued that the choice of the equality of the three corresponding angles gives rise to the development of a new section-similarity of figures.

The choice of this example is dictated by the following questions of the research approach:

1) determination of the equality of geometric figures. A geometric figure is called simple if it can be divided into a finite number of triangles. Based on this definition, the simplest figure is a triangle. In cognitive thinking, it is possible to bring a logical connection between the length of a simple figure and the length of a circle through the limit of the sequences of the perimeters of the corresponding simple figure;

2) analysis of various definitions of the equality of triangles, namely, geometric (coincidence when carrying out a finite number of transformations of motion) and algebraic (equality of lengths and angles).

This example explores the algebraic definition of the equality of triangles;

– the study of the possibility of reducing the number of equalities can also be called the finding of basic equalities, through which it is possible to prove the validity of the remaining equalities;

– exploring the possibility of change in basic equalities. In this case, the accuracy of the basic equalities themselves, as well as their minimality in the system of required equalities, is found out. In addition, the generality of the selected objects is analysed, in particular, when passing to more particular cases, the question of changing large basic equalities is clarified.

Thus, a critical analysis contains the construction of a counterexample that refutes the conclusion of the corresponding statement or leads to the need to prove the preservation of the conclusion. In turn, the search for an example with certain properties constitutes the substantive basis of teaching mathematics at school, as well as in further scientific activities. In some cases, it is possible to move from critical analysis to cognitive thinking.

For the I criterion of equality of triangles, taking: $a=a_1, b=b_1$ and angle $\alpha=\alpha_1$, naturally, it turns out the impossibility of the equality of triangles, two triangles, one of which satisfies the equality, and the other does not, then naturally they will be united by a right-angled triangle. Then, when changing an arbitrary triangle to a right-angled triangle preservation of the result was gotten. It can be also considering the case when the circle intersects the second side of the angle at one point, and the second point lies on the side of the adjacent angle. In such two cases, the angle between the sides can be replaced by another angle, such and other statements may appear after a detailed further study of the object. Analysing the negative result for the conditions $a_1=a_2$, $b_1=b_2$ and the angle $\alpha_1=\alpha_2$, it is necessary to conclude that there are two different triangles with the same general conditions (Figure 2).

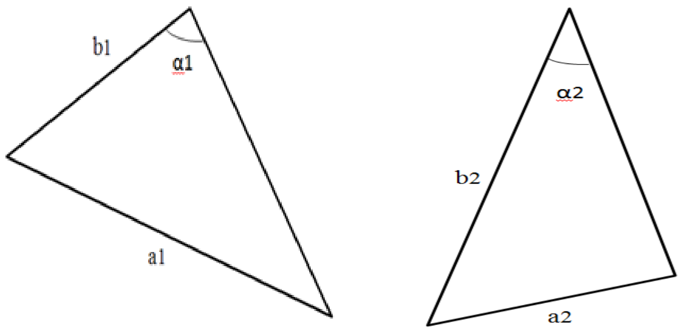

Figure 2. Some types of triangles
Source: [7].

At the same time, if the triangle is rectangular, then the condition $a_1=a_2$, $b_1=b_2$ and triangle $\alpha_1=\alpha_2$ is reduced to the second feature with the additional condition: $a_1=a_2$ or $b_1=b_2$. Such thorough analysis can lead to a fuller understanding of the topic. Scientists from all over the world have conducted research in the field of mathematics education and investigated ways and methods to make it more effective. For example, P. T. Tinh et al. as well as authors of this article, discussed the preparation of teachers for teaching mathematics in primary schools, but in Vietnam [10]. The authors explored the current state of mathematics teacher education in Vietnam and identified areas for improvement. They also described the specific challenges faced by pre-service teachers in preparing to teach mathematics, such as the need to develop deep content knowledge, pedagogical skills, and cultural competence. The article discussed innovative approaches to mathematics teacher education in Vietnam, such as the use of technology, collaboration between teacher education institutions and schools, and partnerships with international organizations. H. Suyitno et al. from Japan discussed how Japanese culture and values, such as national pride, respect for authority, and commitment to social harmony, influenced the way mathematics was taught and learned in Japanese elementary schools [11]. They also explored how these values were incorporated into the curriculum and instructional practices, such as the use of Japanese cultural examples and context in problem-solving activities. Authors also discussed how the integration of nationalism and integrity values into mathematics education in Japan could contribute to the development of students' character and sense of social responsibility, as well as their academic achievement in mathematics. In this article, the influence of national values on the study of mathematics was not studied. However, the authors of this study agree with the authors of the aforementioned research that it is important to take into account various aspects when teaching mathematics.

There were some issues that were not explored in the context of this study, but it is important to discuss them in relation to their significance in teaching mathematics in schools. For example, the study suggested that the use of a competency-based approach, in addition to the research-based approach, should be employed by mathematics teachers what O. V. Berseneva says in her study [12]. The author discussed the concept of competence-oriented tasks and how they could be used to develop the research competencies of future teachers. She also discussed the benefits of using competence-oriented tasks to enhance research competencies, such as increased motivation, improved critical thinking skills, and better understanding of research methodology. It is of no small importance to utilise online courses in the teaching of mathematics, as in recent years the global situation has necessitated a shift to online learning for all. Any mathematics teacher should possess the requisite competencies and skills to conduct online classes. A. Barana et al. examines the methodologies and technologies employed in creating online courses and provides an illustration of the interactive materials integrated into the courses [13]. The outcomes indicate that the resources are valued by educators, mentors, and pupils, suggesting that open online educational resources have the potential to bring about a transformation in Mathematics instruction and learning.

STEM also can be used for teaching mathematics by integrating science, technology, engineering, and mathematics concepts into the math curriculum [14]. This can be done by designing hands-on activities that allow students to explore math concepts through real-world applications. For example, students can use math concepts to design and build models, solve engineering challenges, or analyse data from scientific experiments. This approach can help students see the relevance of math in their daily lives and better understand how it can be applied in various STEM fields. Additionally, integrating technology into math instruction can help students develop important computational and analytical skills, which are essential in many STEM careers. Overall, using STEM in math instruction can help to make math more engaging and relevant for students, and better prepare them for the challenges and opportunities of the future.

The authors of this study agree with M. T. Muanifah et al. that a peer pedagogical approach to learning outcomes in mathematics in primary school is quite effective, as is the research approach. In their study, the authors compared the results of using two approaches in teaching – mutual and group with presentations [15]. To do this, a representative study was conducted with third grade students. As a result, the authors concluded that the use of peer learning in mathematics was more effective than the use of group learning with presentation.

In the study, A. Rijal and A. Azimi focuse on the development of digital teaching materials for elementary school mathematics using whiteboard animation [16]. The authors discuss the importance of technology in mathematics education and how digital teaching materials can improve learning outcomes. They also describe the process of creating whiteboard animation videos and how they can be used in the classroom. The article includes a study on the effectiveness of the teaching materials on primary teacher education students, which found that the use of whiteboard animation improved students' understanding and motivation to learn mathematics. The authors conclude that digital teaching materials, such as whiteboard animation, can be a valuable tool for improving mathematics education in elementary schools. The authors of this study agree with the authors of the aforementioned study that the use of modern technologies is effective for learning, however, this problem was not considered in this research.

The importance and effectiveness of the use of modern technologies is also emphasized in the study by Q. Liu et al., they discuss the development and implementation of a differentiated training program for primary school mathematics education using artificial intelligence (AI) [17]. The authors explain the importance of individualized education and how AI technology can be used to create personalized learning experiences for students. They describe the process of developing the AI-based training program, which includes analysing student performance data to identify individual learning needs and creating customized learning paths for each student. The study also includes a pilot test of the program, which found that students who received the AI-based training showed significant improvement in mathematics performance compared to those who received traditional instruction. The authors conclude that the use of AI technology in primary school mathematics education has the potential to improve learning outcomes and promote individualized learning. In this study, the use of artificial intelligence was not considered, but the authors of this study agree that it can be effective in math classes.

The article by N. Syaifuddin focuses on the implementation of authentic assessment in mathematics teaching among junior high school teachers [18]. The study aimed to explore the practices of authentic assessment in mathematics and the challenges faced by teachers in implementing it. The author used a qualitative research approach with data collected from interviews and observations of six mathematics teachers from two junior high schools in Indonesia. The findings suggest that teachers were generally positive towards using authentic assessment and found it to be more effective in assessing student learning compared to traditional methods. However, teachers also faced challenges in terms of time constraints, lack of resources and training, and resistance from students and parents. The article highlights the importance of providing support and training to teachers in implementing authentic assessment to improve mathematics education.

The article by M. Maximo-Pereira and A. M. Cunha investigates the role of the teacher in developing inquiry-based science teaching [19]. The authors review several studies that explore the impact of inquiry-based teaching on students' learning outcomes and teacher professional development. They also discuss the challenges that teachers face when implementing inquiry-based teaching and suggest strategies to overcome these challenges. The article highlights the importance of teacher training and support in promoting inquiry-based teaching and improving science education. The study by the authors Maximo-Pereira and Cunha also investigated the impact of the research approach on the effectiveness of learning mathematics in elementary school. However, in contrast to this study, the authors' study focuses more on the role of the teacher in this process. In this study, more attention was paid to the components of the research approach.

Based on the foregoing and the examples given, it is possible to formulate a definition of the concepts of “critical analysis” and “cognitive thinking”:

– critical analysis: if there are some restrictions or conditions in the definition or in the formulation of theorems and statements ascertaining the accuracy of the restrictions or conditions, i.e., search for examples confirming statements with the specified conditions, as well as counterexamples refuting the conclusion. Conducting such a critical analysis can lead to a more complete understanding of the topic under study by definition and given properties. A comparative analysis is also important here, examples confirming its importance are given below.

– cognitive thinking: application of generalization of simple tasks, i.e., generalization of some parameters presents in the conditions of a known problem; the development of the topic by replacing the input factors with other factors with some common properties; factors can be the main objects, as well as some changed properties.

The large volume and variety of areas of critical analysis and cognitive thinking as components of the research approach requires from the teacher not only a broad outlook, but also deep knowledge [20; 21]. Therefore, in some tasks, it makes sense to creative cooperation between schools and university scientists. In addition, the implementation of critical analysis and cognitive thinking cannot be permanent; more precisely, it must be used depending on the topics of the educational material themselves. The following division of topics, as well as the physiological characteristics of the students may be the reasons for this statement.

Conclusions

During the study, it was found that the preparedness of a school graduate with an appropriate level of culture of thinking might be a sufficient level of preparedness to study the methods of mathematical modelling, as well as research methods of mathematical science. The application of critical and developmental analysis to some sections can lead to the emergence of a reverse reaction, namely, there may be a need for a deeper study of it for future development. The teacher's actions should be aimed at substantiating the need to study the task at hand, as well as in the correct formulation of the task itself, after which one can proceed to a joint analysis of the mathematical methods involved in solving this task.

Research activities will only be properly organized when skilful guidance from the teacher is carried out, which includes a complete program related to the formulation of the problem, the history of the development of the issue with the results obtained, as well as predicting the expected results. The task itself, allocated for research activities, may be: some generalization of a known theoretical or practical problem; a specific task arising from the comparison of certain known tasks; a new task, for the solution of which it is necessary to combine the solutions of individual tasks. Therefore, the teacher, depending on the level of mathematical education and the intellectual potential of students, should be able to formulate a new problem.

When implementing critical analysis and cognitive thinking, such concepts as the materiality and insignificance of conditions can be clarified, the insignificant conditions of which in mathematical science are called technical conditions, because such conditions arise during the use of a particular research method. In addition, the formulation of new research-type problems can be defined, for some of which there is the possibility of studying in the classroom, while others can be studied in optional and circle classes, and some can also be taken out as topics for students' research work. In further research, it is necessary to study the effectiveness of the application of the research approach not only in mathematics lessons, but also in other school subjects. It is also necessary to conduct empirical research comparing different approaches in mathematics lessons.

Acknowledgements

None.

Conflict of Interest

None.

References

[1] Luiza R, Rabiga K, Amina A, Borashkyzy AU, Uaidullakyzy E, Bakhytgul S. Formation of research skills of students through solving problems in teaching mathematics in primary classes. *Cypriot J Educ Sci*. 2022;17(8):2567-2579.

[2] Zhanys AB, Dautov AO, Aktayeva AU, Askarova AZ. Thinking development and aesthetic education of students in the process of teaching mathematics by example solutions for one problem. *IOP Conf Series: Mater Sci Engineer*. 2020;934(1):012045.

[3] Mussaibekov R. Research approach in the process of teaching mathematics. *Sci Heritag*. 2021;80:51-55.

[4] Zhadyra K, Marat S, Bagdat K. The current state of teaching probability theory and mathematical statistics in secondary schools in Kazakhstan. *Rev Int Geograph Educ Online*. 2021;11(5):2467-2478.

[5] Visnovska J, Cortina JL. Teaching, teachers, and teaching resources in mathematics education research. *Asia-Pacific J Teach Educ*. 2022;50(2):156-164.

[6] Pogorelov AV. *Geometry: A textbook for grades 7-11 of the environment*. Moscow: Prosveshcheniye; 1995.

[7] Smirnov VA, Tuyakov EA.*Geometry. Textbook for the 7th grade of general education*. Almaty: Mektep; 2018.

[8] Mishra L, Gupta T, Shree A. Research-based teacher education from stake holders point: A case study. *Uni J Educ Res*. 2020;8(9):3895-3901.

[9] Toquero CMD. "Real-world:" preservice teachers' research competence and research difficulties in action research. *J Appl Res High Educ*. 2021;13(1):126-148.

[10] Tinh PT, Le TTT, Nguyen P-T, Le CD, Nguyen M-T, Nguyen T-T. Preparing pre-service teachers for mathematics teaching at primary schools in Vietnam. *Eurasia J Math, Sci Tech Educ*. 2021;17(5):em1966.

[11] Suyitno H, Zaenuri Sugiharti E, Suyitno A, Baba T. Nationalism and integrity values in teaching-learning process of mathematics at elementary school of Japan. *J Physic: Conf Series*. 2019;1321(2):022116.

[12] Berseneva OV. Competence-oriented tasks as a means of improving the research competencies of future mathematics teachers. *Inter J "World Sci"*. 2015;3(3):1-17.

[13] Barana A, Casasso F, Fissore C, Marchisio M, Roman F. Mathematics education in lower secondary school: four open online courses to support teaching and learning. In: *18th International Conference on Cognition and Exploratory Learning in Digital Age (CELDA 2021)* (pp. 95-102). London: Springer; 2021.

[14] Nogaybaeva G. Development of STEM education in the world and Kazakhstan. *Educ Count*. 2016;57:20.

[15] Muanifah MT, Rhosyida N, Trisniawati T, Anggraheni R, Maghfiroh N, Kurniasih A, Sa'Diyah H. Reciprocal teaching approach towards mathematics learning outcome of elementary school teacher education students. *J Physic: Conf Series*. 2021;1987(1):012028.

[16] Rijal A, Azimi A. Development of digital mathematics teaching materials in elementary schools using whiteboard animation for primary teacher education students STKIP PGRI lubuklinggau. *J Physic: Conf Series*. 2021;1987(1):012002.

[17] Liu Q, Wang F, Lin F, Yao C, Wang Y. Research on differentiated training program of primary school mathematics education based on artificial intelligence. In: *International Conference on Information and Education Technology* (pp. 268-273). ICIET: Matsue; 2022.

[18] Syaifuddin M. Implementation of authentic assessment on mathematics teaching: Study on junior high school teachers. *Euro J Educ Res*. 2020;9(4):1491-1502.

[19] Maximo-Pereira M, Cunha AM. The teacher who develops inquiry-based science teaching: What do the research say? *Investig Ensin Cienc*. 2021;26(3):134-156.

[20] Kazima M, Longwe J, Gobede F. Improving teaching and learning mathematics in Malawi primary schools: A review of reforms, interventions, successes and challenges. *J Appl Learn Teach*. 2022;5(2):81-89.

[21] Tan P, Padilla A, Lambert R. A critical review of educator and disability research in mathematics education: A decade of dehumanizing waves and humanizing wakes. *Rev Educ Res*. 2022;92(6):871-910.

Про сутність та основні складові дослідницького підходу у навчанні математики в школі

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Анотація

Актуальність. Сучасний етап розвитку суспільства характеризується стрімким прогресом науки, що виражається в глибокому вивченні математичних методів моделювання природних явищ і різних процесів, тобто велика увага приділяється методам математичного моделювання.

Мета роботи. У статті досліджується проблема того, що без відповідного дослідницького підходу у навчанні математики, тобто без організації навчального процесу та впровадження критичного аналізу та пізнавального мислення, неможливо сформувати високий рівень критичного мислення.

Методологія. Основним методом проведення дослідження став метод контент-аналізу, також використовувалися інші загальнотеоретичні методи.

Результати. Для детального вивчення критичного аналізу та когнітивного мислення процитовано деякі роботи дослідників, що стосуються процесу мислення. Разом з тим, дослідження в галузі критичного аналізу та когнітивного мислення проводяться, що підтверджується різними прикладами. На завершення необхідно сказати, що тільки ретельний аналіз може призвести до більш повного вивчення теми. Напрямки критичного аналізу та когнітивного мислення, а також їх взаємодія будуть показані на конкретних прикладах навчального шкільного матеріалу. Зрозуміло, що критичний аналіз і когнітивне мислення потребують розвитку і вдосконалення.

Висновки. Практичне значення дослідження полягає у формуванні специфічних характеристик критичного аналізу та когнітивного мислення в рамках дослідницького підходу.

Ключові слова: критичний аналіз; контрприклад; ознаки рівності трикутників; арифметична та геометрична прогресії; когнітивне мислення.