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Development of effective algorithms for calculating current distribution coefficients of an electric power system

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Abstract. Calculations of steady-state modes of a complex electric power system are significantly simplified if we use the coefficients of distribution of the setting currents. The use of the theory of directed graphs for the formation of the matrix of the coefficients of distribution of the setting currents is directly related to the determination of the values of all possible and specific trees of the graph of a complex electric network. The authors have developed and implemented an improved algorithm. calculation of current distribution coefficients of the electric power system. The concepts of the "base" of the graph and the "hanging" contour are introduced. In this case, to calculate the current distribution coefficients, it is necessary to take into account all possible and specific trees of only the graph base. A method is indicated for supplementing the matrix of current distribution coefficients of the graph base to the current distribution matrix of the original graph. The paper proposes methods for simplifying the network topology based on the development of diacoptic principles.
Key words: electrical network, topology, graph, graph tree, diakoptics, distribution coefficients.

Introduction

The study of electrical networks is always a pressing and complex task. The complexity lies primarily in the large dimension and nonlinearity of the resulting systems of equations that are solved by iterative methods, with all the consequences that arise from this, such as an increase in the number of operations, weak convergence of methods, finding additional restrictions to select the desired solution from a variety of solutions, etc. Calculations of electrical networks are significantly simplified if the distribution coefficients of the setting currents are known [1]. The matrix of distribution coefficients can be formalized both by scheme transformation methods and by analytical methods.

Topological methods for determining the matrix of distribution coefficients were developed in [2]. These methods were associated with the difficulties of searching and determining the values of all possible trees, especially specific 2-trees of the graph of a complex electrical network. In [3], an analytical approach to determining the topological content of the distribution coefficients of the setting currents is presented, which significantly simplified the technology of searching for specific trees of a directed graph of a complex network. This work is a continuation of a series of works devoted to optimizing algorithms for finding all possible and specific spanning trees of a directed graph, with the aim of reducing the number of operations and the amount of RAM on a computer.

Graph theory and topological content of current distribution coefficients

The foundations of the topology of the equivalent circuit of electrical networks were laid in the classical works of Kirchhoff and Maxwell [4,5]. They were the first to introduce the concept of trees and to obtain the topological properties of the determinants of the matrix of nodal conductivities $\underline{Y}$yand the matrix of circuit resistances $\underline{Z}$к.

The current distribution coefficients can be found by solving a system of nodal equations:

$$\underline{C}_{ij} = \frac{Y_{rs}(\Delta_{ir} - \Delta_{js})}{\Delta} \quad (1)$$

where $Y_{rs}$is the conductivity of branch i between nodes r and s.

Based on the study of determinants (1) of nodal equations of a complex electrical network, in [6], the topological content of distribution coefficients was established, which, due to the complexity of search algorithms and the definition of all possible, especially 2-trees of the graph, has not found practical application. In [7], an analytical approach to determining the topological content of current distribution coefficients is proposed. The essence of the approach is that an analytical proof of the formation of specific trees of a directed graph is obtained, without determining 2-trees of the graph, based on all its possible trees [8]. Then, the value of the distribution coefficient in ia branch from the node current $j$is determined by the topological expression:

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$$\underline{C}_{ij} = \frac{\underline{F}_{ij}}{\sum \underline{F}}, \tag{2}$$

where is $\sum \underline{F}$the total value of all possible trees in the graph; $\underline{F}_{ij}$is the algebraic sum of the values of specific trees in the graph of the i -th branch.

Specific trees of the i -th branch relative to the j -th node are formed among possible trees as follows: if the direction of the graph of the i -th branch coincides with the direction of the traversal from the j -th node to the base node, then the tree size is multiplied by +1, otherwise by -1, if the i -th branch is absent, then by 0.

Ways to improve algorithms for finding possible graph trees

In the work [9], an effective algorithm for searching and determining the weights of possible graph trees without using previously defined trees is implemented. The proposed algorithm works successfully, especially in the case of a poorly filled vertex adjacency matrix [10]. With a more filled vertex adjacency matrix, the program execution time increases many times.

For example, for a fairly simple 14-node scheme (http://energy.komisc.ru/dev/test_cases) the number of spanning trees is 3909, and for a 30-node scheme the number of spanning trees is 7,824,000. At the same time, for a 39- node scheme the number of spanning trees is 421,380. That is, each network has its own characteristics. This work is devoted to identifying and taking into account these characteristics.
Let us illustrate it with the following example.

Table 1. Node data. Unom = 220 kV

No	Voltage		Load power		Generation power	
	Phase, deg	module, kV	P, MW	Q, MVar	P, MW	Q, MVar
1	201,9	2,50	0	0	-128	-126
2	214,7	0.99	18	10	0	0
3	198,0	-1,59	20	15	60	35
4	193,5	0,19	0	0	-160	-105
5	199,5	-1,05	13	9	0	0
6	204,8	3,88	94	49,5	0	0
7	209,1	5,87	80	43	0	0
8	207,0	3,57	35	21	0	0
9	212,8	3,77	21	12	0	0
10	211,6	3,08	37	25	0	0
11	205,5	0,36	16	9	0	0
12	112	0	28	14	0	0

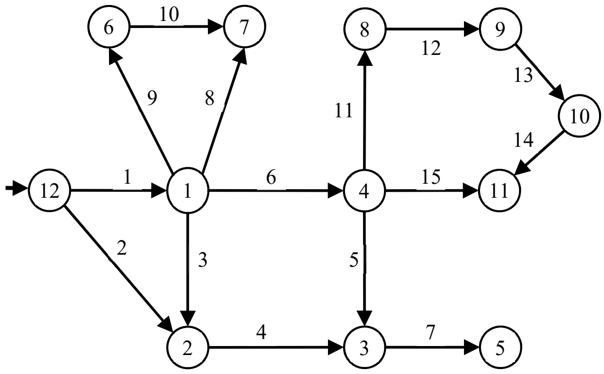

Fig. 1. Network graph.

Table 2. Branch data

No.	Start	End	R, Ohm	X, Ohm	B, mSm (capacity +, ind -)
1	12	1	87,0	88,0	-
2	12	2	14,4	13,0	-
3	1	2	40,0	86,0	-
4	2	3	6,10	71,0	0.8540
5	4	3	39.50	24,0	0,2640
6	1	4	19,00	35,0	0,1372
7	3	5	2,20	30,0	0,3740
8	1	7	3,30	44,0	2,1920
9	1	6	0,60	9,0	0,4320
10	6	7	5,20	73,5	3,6200
11	4	8	6,28	59,4	4,3800
12	8	9	13,00	47,0	0,1490
13	9	10	35,00	13,0	0,3460
14	10	11	14,70	47,0	0,5760
15	4	11	35,00	13,0	0,5760

Using the Kirchhoff matrix, we can calculate that the number of spanning trees is 165.

Recall that the indices of the element of the matrix of current distribution coefficients $\underline{C}_{ij}$denote $i$the -th branch relative to $j$the -th node. Formula (2) can be rewritten as

$$\underline{C}_{ij} = \left(\sum_k G_k \cdot \alpha_{i,j}^k\right)\left(\sum_k G_k\right)^{-1}, \tag{3}$$

here $G_k$is the weight of the tree, calculated as the product of the complex resistances of all the chords $k$of the tree, and the coefficient $\alpha_{i,j}^k$is calculated using the rule:

- $\alpha_{i,j}^k = 0$,if branch j is a chord of the k -th tree.
- $\alpha_{i,j}^k = 1$, if the direction of branch j coincides with the direction in the path from the i-th vertex to the basis vertex in the k -th tree.
- $\alpha_{i,j}^k = -1$, if the direction of branch j is opposite to the direction in the path from the i-th vertex to the basis vertex in the k -th tree.

During implementation, the following stages should be highlighted:

Stage 1. – Identification of all hanging branches. These branches are present in all trees, therefore in formula (3) the complex resistances are reduced and the values of these resistances do not affect the values of $\underline{C}_{ij}$. The coefficients $\underline{C}_{ij}$ will be equal to 1 if j-th node is the beginning of $i$the -th branch, are equal to -1, if $j$the -th node is the end of $i$the -th branch and zero, if $j$the -th node is not incident $i$to the -th branch. And in formula (4) the values $\alpha_{i,j}^{k}$for the terminal j-th node for other branches are equal to the values of such coefficients of the neighboring node q: $\alpha_{i,j}^{k} = \alpha_{i,q}^{k}, \forall i$. Thus, it is sufficient to consider a network without hanging branches. In the network under consideration (Fig. 1) the hanging branch is 7 and the terminal node is 5.

With such "removal" of a hanging branch, other hanging branches may arise. Then the first stage is repeated. In the considered scheme, stage 1 does not lead to the appearance of new hanging branches.

Thus, we need to consider a network with 11 nodes and 14 branches. The number of spanning trees of the network is preserved.

Stage 2. Selection of all "hanging" contours. By a hanging contour we mean a contour that has a single common connecting node with the "base" network, i.e. the remainder of the network after removing the "hanging" contours. In the scheme under consideration, there are two "hanging" contours (Fig. 2).

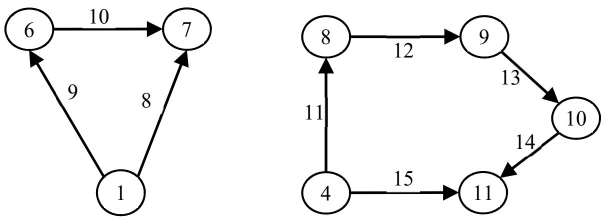

Fig. 2. Two “hanging” contours.

The basis of the graph will be as follows (Fig. 3).

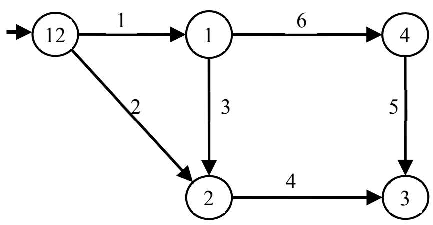

Fig. 3. The basis of the graph.

Table 3. Data on hanging contours

Contour number	Branches of the contour	Contour nodes	The connecting knot
1	8, 9, 10	1, 6, 7	1
2	11,12,13,14,15	4,8,9,10,11	4

Each spanning tree of the network base generates three trees from the first contour and five trees from the second contour. That is, each tree of the network base generates 15 trees of the original graph. So, the network base will be $165: 15 = 11$spanning trees.

Let $\tilde{G}_k$the weight of the tree be the network base, and q_1, q_2let be the connecting nodes of two hanging contours. In our case $q_1 = 1, q_2 = 4$.Then for each tree of the base in the original graph there will correspond 15 trees with weights $G_k$obtained by multiplying $\tilde{G}_k$by a pair of edges, one from each contour, which are respectively a chord of the tree of the original graph. In formula (4), the values $\alpha_{i,j}^{k}$for each j-th node of the hanging contour for the branches of the contour are calculated in the same way as in the graph base, only instead of the base node, a connecting node is taken, and for the branches of the network base in the same way as for the connecting node. In particular, for the graph under consideration, 15 trees will be determined by adding pairs of chords (Table 4).

Table 4. Table of added pairs of chords

Tree number	1	2	3	4	5	6	7
Chord from the first circuit	8	8	8	8	8	9	9
Chord from the second circuit	11	12	13	14	15	11	12

Tree number	8	9	10	11	12	13	14	15
Chord from the first circuit	9	9	9	10	10	10	10	10
Chord from the second circuit	13	14	15	11	12	13	14	15

The values $\alpha_{i,j}^{k}$for all nodes of one circuit and branches of another circuit are equal to zero. The values $\alpha_{i,j}^{k}$for all nodes of each circuit and branches of the network base are equal to the values of $\alpha_{i,j}^{k}$the connecting node of the corresponding circuit. The latter will be known to us, since the connecting nodes are also nodes of the network base. Thus, the current distribution coefficients $\underline{C}_{ij}$for the branches $i$of the base and nodes $j$of the hanging circuit are equal to the values of the current distribution coefficients of the connecting node of the corresponding circuit.

The values $\alpha_{i,j}^{k}$for all nodes of the base and branches of any hanging branches and contours are zero. After the above "cuts" we need to find the weights of the spanning trees and calculate the values $\alpha_{i,j}^{k}$for the base of the graph, which already has 5 nodes and 6 branches, and the number of spanning trees 15, instead of 165 the trees of the original graph. That is, the volume of basic calculations has been reduced by 93.3%.

Let us describe the construction of the matrix of current distribution coefficients of the original network based on the current distribution matrix of the network base (Table 5).

Table 5. Current distribution matrix of the network basis

i \ j	1	2	3	4
1	-0.378-0.056i	-0.097+0.014i	-0.213-0.068i	-0.3-0.039i
2	-0.622+0.056i	-0.903-0.014i	-0.787+0.068i	-0.7+0.039i
3	0.376-0.038i	-0,059+0,009i	0,148+0,066i	0,257-0,021i
4	-0,246+0,018i	0,038-0,005i	-0,639+0,134i	-0,443+0,019i
5	0,246-0,018i	-0,038+0,005i	-0,361-0,134i	0,443-0,019i
6	0,246-0,018i	-0,038+0,005i	-0,361-0,134i	-0,557-0,019i

The current distribution matrix of the complete network will have a dimension of 11×15. It can be constructed in blocks (Table 6). The upper left block will be the same as that of the network base. The remaining elements of the first six rows will be as follows: at column 7, the same as that of column 5, since node 7 is connected to node 5. At columns 6 and 7, the same as that of column 1, and the remaining columns 8,9,10,11 are the same as that of column 4, since node 4 is a connecting node of the second "hanging" circuit with nodes 8,9,10,11. The seventh row, corresponding to the seventh branch, will be zero, with the exception of the 5th element, where the value -1 will be located.

Rows 8, 9, 10 also contain zero elements, with the exception of columns 6 and 7. The indicated 6 elements can be found as the coefficients of the current distribution matrix of the three-node network corresponding to contour 1 (Fig. 2). In this case, node 1, which connects this contour with the base, should be taken as the base vertex . Similarly, rows 11, 12, 13, 14, 15 will have nonzero elements in the columns corresponding to vertices 8, 9, 10 and 11. This block of 20 elements can be found as the current distribution matrix of the five-node network corresponding to contour 2 (Fig. 2). In this case, node 4, which connects this contour with the base, should be taken as the base vertex.

Table 6. Current distribution matrix of the original network

i \ *j*	1	2	3	4	5	6	7	8	9	10	11
1	-0,38- 0,06i	-0,10 +0,01i	-0,21 - 0,07i	-0,3 - 0,04i	-0,21 - 0,07i	-0,38 - 0,06i	-0,38- 0,06i	-0,3 - 0,04i	-0,3 - 0,04i	-0,3 - 0,04i	-0,3 - 0,04i
2	-0,62 +0,06i	-0,90- 0,01i	-0,79 +0,07i	-0,7 +0,04i	-0,79 +0,07i	-0,62 +0,06i	-0,62 +0,06i	-0,7 +0,04i	-0,7 +0,04i	-0,7 +0,04i	-0,7 +0,04i
3	0,38 - 0,038i	-0,06 +0,01i	0,15 +0,07i	0,26 - 0,02i	0,15 +0,07i	0,38- 0,04i	0,38 - 0,04i	0,26 - 0,02i	0,26 - 0,02i	0,26 - 0,02i	0,26 - 0,02i
4	-0,25 +0,02i	0,04 - 0,05i	-0,64 +0,13i	-0,44 +0,02i	-0,64 +0,13i	-0,25 +0,02i	-0,25 +0,02i	-0,44 +0,02i	-0,44 +0,02i	-0,44 +0,02i	-0,44 +0,02i
5	0,27 - 0,02i	-0,04 +0,01i	-0,36 - 0,13i	0,44 - 0,019i	-0,36 - 0,13i	0,25 - 0,02i	0,27 - 0,02i	0,44 - 0,02i	0,44- 0,02i	0,44 - 0,02i	0,44 - 0,02i
6	0,25 - 0,02i	-0,04 +0,01i	-0,36 - 0,13i	-0,56 - 0,02i	-0,36 - 0,13i	0,25 - 0,08i	0,25 - 0,02i	-0,56 - 0,02i	-0,56- 0,02i	-0,56 - 0,02i	-0,56 - 0,02i
7	0	0	0	0	-1	0	0	0	0	0	0
8	0	0	0	0	0	-0,07- 0,00i	-0,65- 0,00i	0	0	0	0
9	0	0	0	0	0	-0,93 +0,00i	-0,35 +0,00i	0	0	0	0
10	0	0	0	0	0	0,07 +0,00i	-0,35 +0,00i	0	0	0	0
11	0	0	0	0	0	0	0	-0,74 +0,12i	-0,51 +0,18i	-0,37 +0,06i	-0,14 +0,12i
12	0	0	0	0	0	0	0	0,26 +0,12i	-0,51 +0,18i	-0,37 +0,06i	-0,14 +0,12i
13	0	0	0	0	0	0	0	0,26 +0,12i	0,49 +0,18i	-0,37 +0,06i	-0,14 +0,12i
14	0	0	0	0	0	0	0	0,26 +0,12i	0,49 +0,18i	0,63 +0,06i	-0,14 +0,12i
15	0	0	0	0	0	0	0	-0.26- 0.12i	-0.49- 0.18i	-0.63 - 0.06i	-0.86 - 0.12i

Of course, for the considered example, the reduction of operations is not very significant for calculations using a computer. However, if we consider a 39- node test scheme, then the number of spanning trees is 421,380, while the graph base contains 22 nodes and 27 branches, and the number of spanning trees is 28,092, instead of 421,380 trees of the original graph. That is, the volume of basic calculations has also decreased by 93.3%, and this is already 393,288 spanning trees less.

Conclusions

The proposed method for identifying the graph base, hanging branches and contours significantly reduces the number of calculations when calculating the current distribution coefficients of the electric power system.

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